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Aristoxenus' Theorems and the Foundations of Harmonic Science

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I Introduction

The third book of Aristoxenus' *Harmonica Elementa*[1] is presented as a collection of theorems, arranged systematically, though without the rigorous precision of Euclid's *Elements* or the *Sectio Canonis*. Aristoxenus' style is more conversational, and he does not preserve all the decorous formalities that we expect of an axiomatic system. In particular, he does not explicitly enunciate, at the outset, the axioms and assumptions on which his reasoning will be based, and the reasoning itself leaves gaps, steps that the reader must full in for himself.

The general intention is nevertheless clear. He announces a series of propositions, and offers for each of them what purports to be a deductive proof. The proofs of later propositions often use as premises propositions that have been proved already. And in the earlier proofs, which need to be derived from more fundamental principles, Aristoxenus to some extent makes up for his failure to provide a preliminary list of axioms by spelling out, in the course of individual arguments, the premises on which they rely. Where he does so, the premises turn out to be principles established in the course of books 1 and 2, on the basis of arguments that are partly dialectical, partly inductive, and are ultimately grounded in assertions about what the ear of an educated musician will perceive as melodically legitimate.

A careful reading of the theorems reveals gaps in the workings of the proofs. These gaps are not always constituted by the mere suppression of inferential steps: often they demand the insertion of additional premises, and it is difficult, in a substantial number of cases, to decide exactly what these should be. It is natural to turn back to books 1 and 2 in the search for principles and assumptions that will do the job. Aristoxenus even offers a hint about where to look, when at the beginning of book 3 a proposition is said to follow ἐκ τῶν ὑποκειμένων (58.24). In that instance he specifies the particular ὑποκειμένον on which he is relying, and we may expect it to be significant that the principle in question appears, with others, in lists of fundamental propositions given in both the previous books, sometimes introduced by the injunction ὑποκείσθω (see especially 29.1–34, and 53.33–55.2). These lists, then, provide the obvious hunting-ground for the missing assumptions of book 3, and it is true that principles capable of supporting the problematic inferences can in several cases be found there.

But this is precisely where the most interesting difficulties begin. In the first place, the ὑποκειμένα suppressed in the proofs cannot always be supplied so conveniently and simply. Second and more important, the assumptions required, whether or not

they have been listed previously, commonly import concepts of an entirely different order from those involved in the principles to which the proofs refer explicitly; and this calls into question the purpose of the entire project.

The apparent objective of the theorems is to establish the order in which intervals of various sizes may legitimately follow one another in a melodic sequence. Aristoxenus is mainly concerned here with melodically incomposite intervals (ἀσύνθετα διαστήματα), that is, with intervals between whose boundaries no note may legitimately fall: by establishing which such interval may follow which he is proving propositions about what we would call 'scales'. (Intervals of sizes that are incomposite in one genus of scale may be composite in another, but the complications introduced by this fact may be ignored for the present.) The point that needs emphasis is that the intervals in question are for the most part identified by their sizes (μεγέθη), that is, as tones, ditones, semitones, and so on, and that the propositions proved are mostly about sequences of intervals of specified μεγέθη (e.g. that one incomposite ditone may not be placed next to another). Correspondingly, the primary principle on which Aristoxenus explicitly calls most often is itself a rule about the compass of the interval that must be occupied by a sequence of lesser intervals. It states that

> any note must either form with the note fourth in order from it (inclusive) the concord of a fourth, or with the fifth in order the concord of a fifth, or both.

Since this pervasive law, which I shall call **L**, treats notes merely as the boundaries of intervals, it could be paraphrased as the rule that if we move from any point on the scale through a series of incomposite intervals, either the first three must sum to the interval of a fourth (which for Aristoxenus is 2½ tones: see 24.4–10, 56.13–58.5), or the first four must sum to the interval of a fifth (3½ tones), or both.

Let us use the word 'quantitative' to refer to the conception of an interval as identified only by the size of the 'space' that it occupies on the continuum of pitch, and to propositions about the properties and relations of intervals conceived in this way. Then it is very easy to receive the impression that Aristoxenus' project in book 3 is to derive quantitative propositions about intervals from equally quantitative premises. If the latter are treated as autonomous principles, and are not derived in their turn from anything else, it will follow that the rules of melodic sequence can be expressed strictly quantitatively and rest on intrinsically quantitative foundations. The laws to be discovered by the science of harmonics would then be expression of the 'natures' of intervals of given sizes, and of the ways in which these quantitative natures can be interrelated.

This impression of what Aristoxenus is attempting is hard to avoid, but it is entirely mistaken. It would indeed be extraordinary if it were correct, given what he says elsewhere about the limitations imposed by a merely quantitative conception of the subject. A consideration of the various magnitudes of intervals as such tells us nothing of any importance in harmonics: once he even claims that the study of mere size constitutes no part of the science at all (40.11–15). In another methodological passage he says that while it is the task of our hearing to grasp magnitudes, by διανοία we study something quite different, which he calls δύναμις (33.6–9). This suggests that a grasp of the sizes of the intervals comprised in a melodic series constitutes a starting-point but not the end-product of the science. They are among its data: we ascend from data to principles (ἀρχαί) and descend again from principles to prop-

ositions that they entail and explain (see 43.25–44.20), but in so doing we have shifted into a new conceptual framework. The propositions to be proved are not about magnitudes as such: hence neither are the principles: and, consequently, the principles themselves are not mere generalisations of the data about the magnitudes that are given to perception. These principles must have a status like that of explanatory hypotheses, perhaps suggested somehow by the quantitative data but not entailed by generalisations from them, and displaying them as consequences of truths of a more fundamental order.

The key concept at the higher level is that of δύναμις. Aristoxenus attributes δύναμις to notes (φθόγγοι), and secondarily also to intervals, defined not by their sizes but by reference to the notes that bound them. Correspondingly, the δύναμις of a note does not depend on the sizes of the intervals between which it is a boundary, nor indeed on its pitch, but on a complex of other factors, most importantly its position in the sequence of notes that constitutes a complete scalar system, a single complete division of melodic space. The concept of melodic δύναμις is a ramified and slippery one: much of what follows is an attempt to explicate it (but see especially section 7). Broadly speaking, however, it may be said to represent the nature or character of a note, expressed in musical practice through the ways in which it is capable of being related to others. A note's δύναμις determines what other notes it can follow and precede, and the melodic relations in which it stands to them: some but not all of these relations are capable of being expressed quantitatively, that is, in terms of the sizes of the intervals that separate one note from another. The crux here is that it is the nature or δύναμις of the note that determines these sizes, and not the other way about.

I shall try to show that the theorems of book 3 are best interpreted as resting on principles of this 'dynamic' order, not on mere generalisations about the sizes of intervals. Their prime objective is not to prove propositions about the possible orderings of such magnitudes: it is to reveal the δυνάμεις of notes, these δυνάμεις being partly, but only partly, expressed in the sizes of the intervals whose boundaries they are. Since the principles must be consistent with inductive generalisations about the magnitudes of intervals, such generalisations are sometimes used as premises in the derivations, and, since some of the δυνάμεις of notes can be expressed in terms of the sizes of adjacent intervals, some of the conclusions are presented in this form. But it is the characters of the notes that determine, unify and explain the data, and Aristoxenus' project is very much one of unified explanation, not simply one of isolated proof.

It would be easy to proceed by pointing to passages in all three books where Aristoxenus makes methodological remarks that support the view I have outlined: some of the relevant passages have already been mentioned, and section 7 will consider them further. But this would hardly be enough by itself to confirm my diagnosis of the puzzles of book 3, if only because an author's theoretical pronouncements about method do not always square with what he actually does. The theorems will have to be worked through in detail. Nor will it be enough to show that it is possible to interpret them from the point of view I am recommending: they might be equally compatible with other readings. In what follows I shall therefore adopt an indirect approach, beginning from the principal alternative to my view, the superficially obvious interpretation that I take to be mistaken. That is, I shall start by seeing how far it is possible to go on the assumption that the propositions Aristoxenus is trying to prove, and the principles from which he seeks to derive them, can all be expressed in terms of the sizes of intervals and their

interrelations. Where this assumption breaks down I shall try to identify the principles with which the quantitative generalisations need to be supplemented and through which they are to be reinterpreted, and in so doing I hope both to establish that propositions about the δυνάμεις of notes are fundamental and indispensable, and to explicate the concept of melodic δύναμις itself.

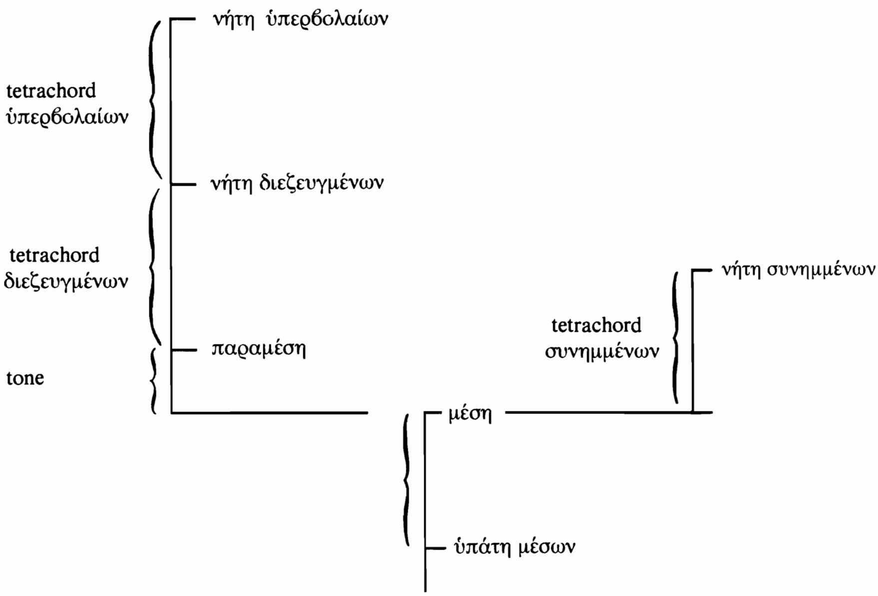
It may be helpful to begin by setting out the more important structural features of the scales that Aristoxenus, as a matter of fact, believed to be melodically legitimate. It is important to emphasize that at this stage it is an open question whether these structures are assumed in advance of the theorems on the basis of educated perception alone or are to be derived deductively from higher principles. The facts, in either case, are briefly these.

First, melodic 'space' as a whole constitutes a two octave continuum. It may be divided into various equally legitimate scalar sequences of intervals, but their variations are limited by a fixed framework of divisions common to them all. This framework forms the boundaries of a series of tetrachords, that is, of sequences of four notes whose outer members span the interval of a fourth. The tetrachords may be linked either in conjunction (συναφή), where successive tetrachords have a note in common (the upper boundary of the lower tetrachord, the lower boundary of the upper), or in disjunction (διάζευξις), where successive tetrachords are separated by an incomposite interval. This interval is always a tone. These rules would in principle allow various different dispositions of the 'fixed' tetrachords within the double octave, but in practice only one is allowed: this arrangement or structure is 'fixed' in the sense of being invariable in all forms of scale, with the one qualification that I shall mention immediately.

Above the fixed note at the bottom of the series (προσλαμβανόμενος) stands another fixed note at the interval of a tone (ὑπάτη ὑπατῶν). This note is the lowest of a tetrachord (the tetrachord ὑπατῶν) whose upper boundary is the note ὑπάτη μέσων, and above this is a tetrachord in conjunction (the tetrachord μέσων) whose upper boundary is the note μέση, an octave above προσλαμβανόμενος. At the interval of a tone above the μέση lies the next fixed note, the παραμέση, disjoining the tetrachord μέσων from the tetrachord διεζευγμένων, whose upper boundary is the νήτη διεζευγμένων. Finally, in conjunction with the tetrachord διεζευγμένων is the tetrachord ὑπερβολαίων, whose upper boundary is the νήτη ὑπερβολαίων, two octaves above the starting point. The system may be schematically represented as follows.

	νήτη ὑπερβολαίων (the highest note)
tetrachord ὑπερβολαίων	
	νήτη διεζευγμένων
tetrachord διεζευγμένων	
	παραμέση
tone	
	μέση
tetrachord μέσων	
	ὑπάτη μέσων
tetrachord ὑπατῶν	
	ὑπάτη ὑπατῶν
tone	
	προσλαμβανόμενος (the lowest note)

The qualification to be made is that as one proceeds up the series there is an alternative continuation from the μέση. Instead of moving to a tetrachord in disjunction, one may continue to a tetrachord in conjunction, the tetrachord συνημμένων. If we proceed by this route, the sequence is usually conceived as ending at the top of this tetrachord without continuing to the complete double octave. Above the μέση, then, the series splits into a pair of alternatives: this fact will be of importance in the theorems. The divided sequence can be set out in the following way.



Within the boundaries of each tetrachord, in any form of the scale, two further notes are located. These are 'movable' notes: the intervals that they form with one another and with the bounding notes of the tetrachord differ in what Aristoxenus calls different genera (γένη) of scale. There are three genera, the enharmonic, chromatic and diatonic, and each of the latter two occurs in several distinct and well defined variants (χρόαι). In fact, according to Aristoxenus' conception of the matter, the positions inside the tetrachords, admit of an indefinite number of subtle variations within any genus. Though the point at which a shift involves change of genus is determinate, each genus may appear in an indefinite number of different χρόαι. This becomes important in the theorems, but here it will be useful to set out three of the tetrachordal divisions that Aristoxenus treats as familiar, as paradigm cases, one in each genus. Since in any one variation of any one genus every tetrachord between fixed notes has the same form, differences of χρόα and genus can be represented as different divisions of any one such tetrachord: Aristoxenus generally takes the tetrachord μέσων as his standard example. (The notes of this tetrachord, reading from the bottom, are called ὑπάτη μέσων, παρυπάτη μέσων, λιχανὸς μέσων and μέση.)

I shall use the following abbreviations: q = quarter-tone, s = semitone, t = tone, d = ditone, other intervals being expressed as fractions of the tone.[2] In the enharmonic

genus the three intervals within the tetrachord, beginning with the lowest, are *q q d*; in one form of the chromatic (the tonic chromatic) they are *s s 3t/2*; in the commonest form of the diatonic (the σύντονον or sharp diatonic) they are *s t t*. In tetrachords bounded by fixed notes these intervals always occur in the same order: between such boundaries the enharmonic sequence *q q d*, for example, cannot be replaced by *d q q* or *q d q*.

These divisions of the tetrachord allow us to fill in the gaps in the two-octave system in any of three ways: an indefinite number of others is permissible, as we have seen, though the fixed framework must always be preserved. We may now proceed to the analysis of Aristoxenus' theorems, bearing in mind that it remains to be decided how much of the structure set out above is assumed in advance of the proofs, and how much is to be demonstrated.

II Sequences of tetrachords

The first propositions in the book concern the ways in which tetrachords may be linked to form sequences. One proposition is treated as fundamental, but several others are set out, with supporting arguments, in response to a collection of ἀπορίαι, puzzles or difficulties (these may perhaps be questions raised in discussion by Aristoxenus' students[3]).

The initial proposition (58.14–59.5) gives little difficulty. It asserts that sequences of similar tetrachords must be either conjunct or disjoined by the interval of a tone. Two points of terminology need explaining. Tetrachords are similar (ὅμοια) if they contain intervals of the same sizes in the same order. They are in sequence (ἑξῆς) if either the upper boundary of the lower is identical with the lower boundary of the upper (i.e., they are conjunct), or the lower boundary of the upper is in sequence (ἑξῆς) with the upper boundary of the lower. This definition is given at 59.13–19, in response to one of the ἀπορίαι: the circularity of its latter part is resolved at 60.10–61.4, where it turns out that two notes are ἑξῆς if the interval separating them is melodically incomposite (ἀσύνθετον), that is, if it is such that in the given form of scale no note can fall between its boundaries.

The proposition asserts, then, that similar tetrachords that either share a boundary or are separated by a melodically incomposite interval must be either conjunct or disjoined by a tone. It is said to follow ἐκ τῶν ὑποκειμένων (58.24), and the relevant ὑποκειμένον is specified: it is the law mentioned in section 1 above, which we are calling **L**. From **L** the proposition does indeed follow. If the tetrachords are conjunct, the fourth note in order from a given note (inclusive) will stand to the given note at the interval of a fourth. If they are separated by a tone, the fifth note in order will stand at a fifth. If they are separated by anything else, neither condition laid down by **L** will be fulfilled.

Problems begin with the group of propositions whose enunciation is stimulated by the ἀπορίαι. The ἀπορίαι are listed at 59.5–12. In response Aristoxenus first gives the definition of sequence (τὸ ἑξῆς) as it applies to relations between συστήματα (groups of intervals, segments of scales), which I have already mentioned, and goes on to state five further propositions (59.19–60.9).

(i) Tetrachords that are ἑξῆς in the first sense, i.e. conjunct tetrachords, must be similar (59.19–23).

(ii) Tetrachords that are ἑξῆς in the second sense, i.e., those disjoined by a

melodically incomposite interval, are similar if the intervening interval is a tone (59.23–27).

(iii) Tetrachords that are ἑξῆς in the second sense cannot be similar if the intervening interval is anything other than a tone (59.26–27). (This proposition with (ii) is equivalent to part of the initial proposition of 58.14–59.5 which is reaffirmed at 59.27–33.)

The fourth and fifth propositions involve what seems to be a new sense of ἑξῆς: it apparently means 'belonging to the same sequence or scale' without implying 'adjacent', though Aristoxenus does not explain it.

(iv) Similar tetrachords that are ἑξῆς can be separated by a tetrachord, but only by one that is similar to them (60.2–4).

(v) Tetrachords that are ἑξῆς but dissimilar cannot be separated by any tetrachord (60.4–6).

The only principle explicitly adduced in support of these propositions is **L**. To this, along with the definitions of ἑξῆς and ὅμοιος, we may add the simple fact that the only subject under discussion is sequence of tetrachords. Aristoxenus does not commit himself here to the thesis that all extended scalar sequences must be analysable as strings of tetrachords: he is simply not talking about anything else at present. It remains to be seen whether the thesis will in the end have to be assumed after all.

It can be shown very simply that propositions (i), (ii) and (v) do not follow from **L** and the immediate context. If we fill in the double octave that constitutes melodic space with a legitimate series of diatonic intervals, we get *t s t t s t t t s t t s t t*. This is the 'correct' sequence as Aristoxenus understands it, and if we assume that the boundaries of the tetrachords are the fixed notes, it must be divided in the pattern *t*, *s t t*, *s t t*, *t*, *s t t*, *s t t*. This sequence obeys all Aristoxenus' rules. If, however, we ignore the fixed notes (which have not been mentioned in the argument), several other analyses into tetrachords become possible, for instance *t s t*, *t*, *s t t*, *t s t*, *t*, *s t t* or *t s t*, *t*, *s t t*, *t s t*, *t s t*, *t* or *t*, *s t t*, *s t t*, *t s t*, *t s t*, *t* or *t s t*, *t s t*, *t*, *t s t*, *t s t*, *t*, and so on.

All of these divisions yield sequences of tetrachords, in conjunction or disjoined by a tone. None of them breaches **L**. But they give counter-examples to proposition (i), since they admit the sequence *s t t*, *t s t*, tetrachords in conjunction but dissimilar; to proposition (ii), since they admit *t s t*, *t*, *s t t*, tetrachords that are dissimilar but separated by a tone; and to (v), since they admit *s t t*, *t s t*, *t s t*, where dissimilar tetrachords are separated by a tetrachord.

It is clear that either something has gone seriously wrong, or else Aristoxenus is making assumptions that he has not announced. The obvious move would be to insist that by 'tetrachord' he means 'tetrachord bounded by fixed notes', and that he is therefore presupposing the framework of fixed notes set out in section 1. But this will not do, in view of the plain implication of (iii) and (v) that dissimilar tetrachords can appear in the same sequence. It is guaranteed by **L** that all tetrachords between fixed notes in the same sequence are similar. A sequence such as *t s t t s t t t s t t s t t* contains dissimilar tetrachords only in the sense that between e.g. the second and fifth notes there is a tetrachord of the form *s t t*, while between the third and sixth there is one of the form *t t s*. But the third and sixth are not fixed notes: fixed notes in this sequence form boundaries to tetrachords of no form but *s t t*. Hence by 'tetrachord' Aristoxenus cannot mean 'tetrachord between fixed notes'.

I shall not attempt to resolve the difficulty for the present: it will be more con-

venient to postpone it until it can be related to problems occasioned by later propositions. Here I shall give only a bald statement of two points which will be developed in the sequel. First, the eccentricity of the alternative readings of the diatonic series depends, from one point of view, on 'improper' placings of the disjunctive tone. If grounds can be given for treating these placings as unacceptable, the errant readings can be eliminated. The location of the disjunctive tone comes under prolonged scrutiny at a later stage. Secondly, these problems can arise only in the diatonic series, not in chromatic or enharmonic, since it is only in diatonic that the interval of a tone appears ambiguously, in two roles: it may be an interval within a tetrachord or the interval by which tetrachords are disjoined. The difficulty would therefore be resolved if it were possible to argue from what is true of the non-diatonic genera to propositions about 'equivalent' stretches of the diatonic series. The possibility will have to be explored: the notion of 'equivalence', however, is not altogether a straightforward one.

III Changes of genus

As a preliminary to the propositions about sequences of incomposite intervals, Aristoxenus undertakes to show (61.5–34) that in changes of genus it is only the 'parts of the fourth' that alter. This amounts to the claim that once the constituent small intervals of a given fourth have been determined, the genus of the whole system of which that fourth is a part is fixed: no further, independent alterations are possible, and hence the analysis of any genus of the two-octave system can be represented by the analysis of any one of its constituent fourths. The proposition is a good example of one that is adopted as an aesthetic datum in books 1 and 2, but which Aristoxenus will now seek to demonstrate.

His argument begins from the thesis that every melodic complex (πᾶν τὸ ἡρμοσμένον) if it is constituted by more than one tetrachord, is divided up into tetrachords by conjunction and disjunction, where by 'disjunction' he means 'disjunction by a tone' (61.14–15). Now if it is assumed that every extended sequence is analysable into tetrachords, this proposition will follow straightforwardly from **L**. If it is not, however, either Aristoxenus is again limiting his remarks to sequences that *are* so analysable, in which case we must ask why he ignores other possibilities, or his proposition will fail to follow from **L**, and will demand further premises in its support.

That Aristoxenus restricts his remarks to scales analysable into tetrachords can be shown without difficulty. If we admit the possibility that an extended sequence need not be divisible into tetrachords, it must still be consistent with **L**. It can be so only if it is divisible into sub-sequences comprising similar pentachords, each spanning a fifth. Examples would include repetitions of *q q q 11t/4* or of *s s s d* or of more bizarre forms such as *s 3t/4 3t/4 3t/2*. Each of these is consistent with **L**, but their repetitions do not generate sequences of tetrachords, since no note will stand to the note fourth in order from it at the interval of a fourth. Taken by itself, **L** implies the possibility of analysing sequences into tetrachords or pentachords or a combination of both: it does not exclude a purely pentachordal system.

The assumption that analysis into tetrachords is always possible is so pervasive in Aristoxenus that it is a plausible candidate for being an initial datum, a generalisation from experience or an underived methodological principle. On the other hand, it is not stated in any of the lists of ὑποκείμενα, and there is in the lists one principle that can

be interpreted so as to allow the necessity of tetrachordal structure to be derived. But it brings its own problems with it.

Book 1 ends with a list of theses each of which is introduced by the word ὑποκείσθω (29.1–33). Some of the theses are merely definitions, but four are more substantial. The first is the one I am concerned with here: let us call it **P**. (The second is **L**, the third is derivable from **P** and **L**, and the fourth seems incapable of doing any work that **L** could not.)

P states that

> once there is a σύστημα, whether it is πυκνόν or ἄπυκνον, it must be followed in the upward series by not less than the remainder of a fourth, in the downward series by not less than a tone (29.1–6).

A σύστημα is a series of intervals, the smallest possible σύστημα being constituted by two intervals in sequence. By 'once there is a σύστημα' ('once a σύστημα is τιθέμενον') Aristoxenus apparently means 'as soon as we go beyond a single interval to a minimal σύστημα', i.e., 'when there is a pair of intervals in sequence'. A πυκνὸν σύστημα, often simply τὸ πυκνόν (πυκνόν has roughly the sense 'compressed'), is a pair of successive intervals which together span an interval smaller than the remainder of a fourth: the pair therefore counts as a πυκνόν if it totals less than 5/4 tones (24.11–14). Otherwise it is ἄπυκνον.

If we consider the sequences of intervals that Aristoxenus standardly treats as legitimate, at least part of **P**, taken strictly, is false. In the enharmonic sequence *q q d*, *t*, *q q d*, there are πυκνά of the form *q q*, and of them **P** is true. But there are also ἄπυκνα συστήματα, for instance those of the form *q d*, and of these **P** is false, since one may proceed downwards to *q*, which is less than the interval of a tone.

Now this comment may fairly be construed as a quibble. In the context Aristoxenus plainly means the σύστημα in question to be either a πυκνόν, or, in those forms of scale that do not contain πυκνά (i.e., all diatonic scales), the pair of intervals that stands in the equivalent position in the tetrachord. This reading of his intentions is certainly correct, but it raises once again the difficult issue of 'equivalence' between segments of generically different scales. I shall turn to this problem shortly: let us first see how **P** can be pressed into service if it is taken to apply only to scales that contain a πυκνόν. Since the sense of that term is defined quantitatively, it can figure in the purely quantitative kind of analysis that we are attempting to pursue.

Taken together with **L**, **P** can be used to guarantee that any extended scale containing a πυκνόν is analysable as a series of similar tetrachords, in conjunction or disjoined by a tone. It follows from **L** that if a scale is not analysable as such a series, it must be a sequence of similar pentachords. If it can be shown that any pentachord in a sequence must contain at least one incomposite interval of a tone, then every series of pentachords will be analysable as a series of tetrachords disjoined in the appropriate way.

In a series of similar pentachords, any set of four consecutive intervals will sum to 3½ tones (by **L**). By **P**, any πυκνόν must have at least a tone below, and at least the remainder of a fourth above. Let *W X Y Z* be any series of four intervals summing to 3½ tones, and let *X Y* be a πυκνόν. Then *W* is at least a tone, and *X Y Z* is at least 2½ tones. But if *W* is at least a tone and *W X Y Z* is 3½ tones, plainly *X Y Z* cannot be more than 2½ tones: conversely, *W* cannot be more than a tone. Hence *W* is a tone and *X Y Z* is a fourth. Each pentachord in a sequence containing πυκνά must therefore contain a tone and a tetrachord spanning a fourth.

For sequences involving the πυκνόν, **P** has a further use: it ensures that the interval of a tone must stand immediately below the πυκνόν: the sequence *d q q*, *t*, *d q q* is not permissible. A tetrachord of 2½ tones containing a πυκνόν cannot itself contain an interval of a tone, since a tone cannot be part of the πυκνόν, given that *q* is the least of the melodic intervals (see e.g. 47.1–2) and that a πυκνόν totals less than 5/4 tones, and since it cannot be the remaining interval because a πυκνόν must occupy less than half the interval of a fourth. Hence the position below the πυκνόν, and outside the tetrachord, is the only one available to the tone in such a system.

The argument to show that every legitimate extended sequence is a string of tetrachords can be generalised to scales not involving the πυκνόν if, but only if, we assume that they contain repetitions of some pair of intervals that counts as the ἄπυκνον σύστημα to which **P** refers. If we do not make this assumption, some eccentric pentachordal schemes will remain untouched by **P**: it cannot be shown, for example, that *3t/4 3t/4 3t/4 5t/4* or *s 3t/4 3t/4 3t/2* is improperly formed.

We have seen that the term πυκνόν can be defined purely quantitatively (though only by its limits, not as one uniquely specifiable μέγεθος). But it will be possible to define 'equivalent to the πυκνόν' for series that contain no πυκνά only by invoking either the notion of order within a complete system, or by calling on the framework of fixed notes outlined in section 1. The former looks more economical, at a superficial glance: if there is a πυκνόν above, say, the second and fifth notes of the enharmonic system, there must be an 'equivalent' in a system without a πυκνόν, above the notes in the same positions. But this is a very inadequate notion of equivalence unless the notes or the melodic 'spaces' demarcated by them correspond to one another also in some more significant way: we are bound to offer some account of what this significance amounts to. Besides, the application of **P** to systems containing a πυκνόν cannot by itself ensure that the two-octave system envisaged by Aristoxenus is the only correct one. It ensures that any system must be composed of tetrachords in conjunction or disjunction by a tone, but it does nothing to show that disjunctive tones are to be placed only at the bottom of the series and immediately above μέση. They might stand between all the tetrachords, or any, or none. Again, suppose that an adequate definition of 'equivalent of the πυκνόν' can be given for systems containing no πυκνά. Then diatonic systems too will be divisible into conjunct and disjunct tetrachords. Suppose further that we can find grounds for eliminating such sequences as *s t t t s t t s t t t s t t* (though this conforms perfectly well to **L** and **P**), and are left only with *t s t t s t t t s t t s t t*, the Aristoxenian two-octave series. We still have no grounds for insisting that it is only the first and the eighth intervals that count as disjunctive tones between tetrachords, thus generating tetrachords only of the form *s t t*: there is nothing yet to rule out such 'improper' readings as *t s t*, *t*, *s t t*, *t s t*, *t*, *s t t*, since **P** tells us only that below the πυκνόν-equivalent (presumably *s t*) there must be at least a tone, and above it at least the remainder of a fourth. These conditions are satisfied. It does not say that only the tone below the πυκνόν-equivalent can count as a tone of disjunction. To guarantee that, we must either assume that disjunctive tones as well as πυκνά and their equivalents stand in the same positions in every genus, or else adopt an independent rule about the form of tetrachord that can legitimately be disjoined from another.

Aristoxenus states what amounts to a weak and qualified version of such a rule at 52.8–11 and 26–34. Of the intervals of the tetrachord, that between the ὑπάτη and παρυπάτη is either equal to that between the παρυπάτη and λιχανός or smaller than

it: the latter interval may be equal to that between the λιχανός and μέση, or smaller, or greater. (The four notes of the paradigm tetrachord are ὑπάτη, παρυπάτη, λιχανός and μέση, reading from the bottom.) In fact the middle interval can be greater than the highest only in certain non-standard (though legitimate) sequences, involving intervals drawn from more than one recognised genus or χρόα. If we ignore this possibility, and focus only on what are for Aristoxenus the normal and familiar kinds of tetrachord, we have a rule according to which

> the lowest interval is smaller than or equal to the second, and the second is smaller than or equal to the highest.

There can then be no interval in such a tetrachord smaller than the lowest: hence a diatonic tetrachord must take the form *s t t*, not *t s t* or *t t s*. Let us call this rule **R**.

However, **R** is stated in terms that refer to named notes of the double octave system. Of those it mentions, the ὑπάτη and μέση are fixed notes: and unless it is assumed that disjunction between tetrachords occurs only between fixed notes, **R** cannot eliminate improper readings of the diatonic series. Without that assumption, the sequence *F t F s t t F s t t* at the bottom of the series (where *F* marks the positions of fixed notes), could still be read as *t s t*, *t*, *s t t*. What is wrong with this reading is that not all the boundaries of the tetrachords it marks out are fixed notes. **R** itself is not broken, since the tetrachords that *are* bounded by fixed notes continue to be of the approved form, *s t t*. And of course, if we assume that only tetrachords bounded by fixed notes can be disjoined, and that these fixed notes stand in the same relations (in terms of size) in every genus of scale, it is still an open question where they lie. The three major principles that we have identified, **L**, **P** and **R**, are not capable of reducing the number of possible fixed frameworks to one: if only one is legitimate, that fact must apparently be assumed in advance.

P and **R** suffer from a further disadvantage, though it can perhaps be expressed only rather impressionistically. Suppose it is true that πυκνά and πυκνόν-equivalents always have at least a tone below and at least the residue of a fourth above: suppose also that the smallest interval in a tetrachord between fixed notes always stands at the bottom. These propositions would nevertheless seem most disappointing, indeed alarmingly *ad hoc*, if given the status of primary principles of explanation. They have the unmistakable smell of inductive generalisations from perceptual experience: they present a pair of uncoordinated facts whose truth surely requires explanation in the light of a unifying principle. Aristoxenus insists that the harmonic scientist must be able to distinguish between what is prior and what is derivative (43.34–44.1), and that the first principles must be recognisably of the right sort to stand at the head of the system (οἷον ἐν πρώτοις ὑπὸ τῆς αἰσθήσεως συνορᾶσθαι τῶν τῆς ἁρμονικῆς πραγματείας μερῶν, 44.11–13): it is not appropriate to a primary principle to be the sort of thing that requires explanation or demonstration (ἀπόδειξις) (44.14–15). **P** and **R** seem to me, at least, signally to fail the tests that Aristoxenus' remarks imply.

Let us turn now to the second part of the argument about changes of genus (61.11–34), allowing it to be assumed that every extended series of intervals is a set of tetrachords in conjunction or disjunction. The proposition to be proved is that in changes of genus, only the 'parts of the fourth' alter: the proof now proceeds in two steps.

(a) The conjunct series contains only parts of the fourth, so that in this case necessarily only these alter in changes of genus (61.11–14).

(b) The disjunct series also contains the tone, so that the question is whether or not it can be altered. But since its boundaries are the bounding notes of tetrachords, and since it has been shown that these bounding notes do not move in changes of genus, it follows that the (disjunctive) tone is invariable. Hence only intervals within disjoined tetrachords (the parts of the fourth) can be altered, not the disjoining intervals themselves (61.14–34).

The first step is unproblematic. The central claim of the second, that the boundaries of tetrachords have been shown to be immoveable, may be intended as a reference back to (a), but in any case the point seems to be that if these notes did move, the tetrachord whose boundaries they are would no longer span a fourth.

Construed in this way, the argument is mildly puzzling. It seems to leave open the possibility that one or other of the tetrachords bounding the tone could be moved up or down as a whole, so stretching or compressing the disjunctive interval. Of course this is impossible, since it would involve a breach of **L**, but **L** is not explicitly appealed to. (**P** is inadequate: it entails that the tone cannot be compressed, but not that it cannot be expanded.)

Aristoxenus' failure to appeal to **L**, here and in several later cases where it could readily have been used, is surprising and hard to explain. Presumably he has something equally fundamental and straightforward in mind, but no other appropriate rules of a purely quantitative sort are available. He might be assuming that changes of genus are always constituted by alterations of exactly the same form whether the tetrachords involved are conjunct or disjunct. In that case they could not involve variations in the interval of disjunction: but the assumption seems to beg the question at issue. Alternatively, his presupposition might be that notes fixed, in every genus, in relation to others in their own tetrachord, are also fixed in relation to all other such fixed notes in the system. In that case, if the disjunctive interval is bounded by fixed notes, and if it is sometimes a tone, it is always a tone.

If this second suggestion is on the right lines, Aristoxenus is assuming, and not setting out to prove, that any extended system is not merely constituted by a string of tetrachords, as **L** and **P** dictate, but is shaped by a skeleton of fixed notes which are themselves the boundaries of tetrachords. More precisely, the whole of any system must be constructed around a series of fixed notes that comprise the boundaries of successive (ἑξῆς) tetrachords. By **L**, tetrachords that are ἑξῆς may be conjunct or disjoined by a tone: hence disjunctive tones may lie between the tetrachords bounded by fixed notes. They can be inserted nowhere else, since to place them inside a tetrachord would involve stretching the tetrachord beyond its fixed limits.

In the context, this assumption must be interpreted to mean that the fixed structure is exactly the same in every genus: a new genus does not import a new skeleton. Since the disjunctive tone is a part of this structure, we can also stipulate that by a 'disjunctive tone' we mean the tone that separates tetrachords whose boundaries are the same in every genus: Aristoxenus sometimes refers to it as 'the tone common to the genera' (e.g., 68.7). In that case we can eliminate variant readings of the legitimate diatonic series *t s t t s t t t s t t s t t*. Since in enharmonic, for instance, the equivalent structure is *t q q d q q d t q q d q q d*, and the fixed structure must be the same in both genera, only the first and the eighth intervals of the diatonic series can count as disjunctive tones.

Two major problems remain. First, though the structure of fixed notes implied by the sequences given above is the only one Aristoxenus allows, there is nothing so far in our principles and assumptions to ensure that it is this one, and not some other, that is the unique legitimate structure. Nothing has shown that it might not be the structure implied, for example, in *s t t*, *t*, *s t t*, *s t t*, *t*, *s t t* and its enharmonic equivalent *q q d*, *t*, *q q d*, *q q d*, *t*, *q q d*. In fact no principles available to Aristoxenus could possibly do the job. We are forced to conclude that the assumption at work behind the present argument is not merely that a fixed structure of some one appropriate kind exists, but that it is precisely the one set out above in the course of section 1. Though many features of this structure can be shown to follow from principles that Aristoxenus explicitly adopts, and most notably from **L**, others remain obstinately underivable and must simply be taken as given. Aristoxenus' arguments depend, then, on a complex assumption which may fairly be reconstructed as follows. Every legitimate scalar sequence must be analysable as a string of successive tetrachords (or as a single tetrachord, or as part of one) whose boundaries are fixed notes: there is one and only one set of fixed notes, those that have been assigned the names προσλαμβανόμενος, ὑπάτη ὑπατῶν, ὑπάτη μέσων, and so on: the intervals between these notes are those given in the diagrams in section 1, and can never be anything else. The three clauses should not be detached from one another: they form a single, though rather involved assumption, and I shall call this assumption **A**.

Secondly, it does not appear to follow from **L** and **A** alone that all tetrachords within fixed boundaries must have, in each genus, the pattern of intervals that Aristoxenus attributes to them, *q q d* rather than *d q q* or *q d q* in enharmonic, *s t t* rather than *t t s* or *t s t* in diatonic, and so on. To generate this conclusion we need something at least as strong as **P**, but the status of **P** is suspect, as we have seen. However, in later propositions of book 3 Aristoxenus makes strenuous efforts to prove that the incomposite intervals cannot appear in 'improper' sequences: we shall seek to identify the principles on which these proofs rest.

Before we reach them, there is one further preliminary proposition to be considered: it need not detain us long. 'In each genus there are at the most as many incomposite intervals as there are (intervals) in the fifth' (62.1–2). The argument depends on the earlier thesis that all extended sequences are formed from tetrachords in conjunction or disjunction by a tone, and the proposition amounts to the claim that in any single genus the incomposite intervals in any tetrachord are of the same sizes as those in any other.

If 'S is a scale in a single genus' entails 'every tetrachord in S has intervals of the same sizes as does every other' just by virtue of the meaning of the expression 'in a single genus', then the proposition is trivially true. If this is not the case, it is nevertheless derivable from the earlier thesis that all tetrachords in conjunction, or disjoined by a tone, are similar. We saw in section 2 that this thesis is not derivable from **L** alone. On the other hand, the errant readings of the diatonic series that cause the problem are eliminated as soon as we adopt **A**: and in any case it does follow from **L** that no tetrachord in a series of successive (ἑξῆς) tetrachords can contain an interval of a size that is not present in all the others, even if the intervals could appear in different tetrachords in different orders.

Then 'S is a scale in a single genus' can be allowed to mean simply 'S is a single continuous system'. By **L** it will follow that no tetrachord contains intervals of sizes

that other tetrachords lack. If all the intervals of a given tetrachord are of different sizes, and if none is a tone, then the series contains the three sizes of interval found in its tetrachords, and the tone of disjunction. Hence it contains at the most the same number of incomposite intervals as there are intervals in the fifth (i.e., four): it may contain fewer, since two or three of the intervals may be the same size.

IV Sequences of incomposite intervals

Though **L** and **A** yield a tetrachordal structure with conjunctions and disjunctions by a tone, and guarantee that every tetrachord within a single system has the same form so long as it is bounded by fixed notes, they do not determine the order in which the intervals within the tetrachords are arranged. The propositions that follow give an exhaustive analysis of permissible sequences of melodically incomposite intervals, and argue that all others are in breach of established principles. Much of the reasoning, once again, is strictly quantitative, but not all of it can be so construed: some of the arguments raise difficulties of a kind that we have not so far encountered. I shall list the propositions in order, and comment on each in turn.

62.34–63.5 A πυκνόν cannot be followed either by another πυκνόν or by a part of one.

This is said to follow from **L**, and **L** does indeed rule out successions of complete πυκνά. By itself, as we saw previously, it does not rule out pentachordal sequences such as *q q q 11t/4*, *q q q 11t/4*, to reject which we need one or other of the principles that guarantee the possibility of analysis into tetrachords. It will be most economical to assume **A**.

63.6–20 The lower of the notes bounding the ditone is the highest note of a πυκνόν, and the higher is the lowest note of a πυκνόν.

The reasoning is as follows. In conjunction, two πυκνά must be concordant at the fourth, and hence there must be a ditone between them. Two ditones must also be concordant at the fourth, and hence there must be a πυκνόν between them. Hence ditones and πυκνά must alternate in the series.

The argument is a straightforward application of **L**, since no notes of two conjoined tetrachords that contain πυκνά can be concordant at the fifth. But there are points to be noticed. First, Aristoxenus states this argument and several that follow in terms relating to the enharmonic series, but they apply equally, *mutatis mutandis*, to any series where tetrachords contain πυκνά. The reference will not then be to the ditone, but to whatever interval it is that makes up the residue of the fourth. Secondly, the notion of one interval, e.g., a ditone, being concordant with another may seem slightly opaque; but Aristoxenus means no more than that each bounding note of the one ditone is so related to its counterpart in the other. In the case of the πυκνόν there are three pairs of notes to be so related, rather than two. This introduces the third point, which is that Aristoxenus seems to be treating the two intervals of the πυκνόν as jointly constituting a single unit whose parts cannot be detached from one another: at 67.15–16 he even refers to the πυκνόν as one of the ἀσύνθετα. However, the proof will stand without this assumption: even if we admit a tetrachord of the form *q d q*, a series of such tetrachords in conjunction will still generate an alternation of ditones and πυκνά, since quarter-tones will stand together in pairs. The πυκνόν will be incontrovertably split only if such tetrachords are disjoined as in the series *q d q*, *t*, *q d q*, and

Aristoxenus later offers arguments to show that such a series is unacceptable.

Finally, it is most important to notice that while the proof refers only to tetrachords in conjunction, the proposition itself is stated quite generally, without that restriction. It is sometimes used as a premise in later arguments in the context of disjunct sequences (e.g., 65.31–66.8), and this fact yields one of the most serious methodological difficulties that the theorems present. Aristoxenus' treatment of the disjunctive tone itself, in the next proposition and a number of its applications, raises problems of a similar kind.

This proposition about the ditone is of sufficient importance in the sequel to merit its own reference-letter: let us call it **D**.

63.21–33 Each of the notes bounding the tone is the lowest note of a πυκνόν.

As grounds for this proposition, Aristoxenus offers the assertion that in disjunction the tone is placed between tetrachords of such a kind that their bounding notes are the lowest notes of πυκνά, since one is the highest note of a tetrachord, the other the lowest.

It is plain that Aristoxenus is considering the tone only in its role as the interval of disjunction. Since the tone can have no other role in sequences whose tetrachords contain πυκνά, this raises no immediate problems, so long as Aristoxenus restricts his focus to such sequences. Difficulties will appear only when the argument is generalised to diatonic sequences (68.2–12, discussed in section 6).

The argument evidently assumes that the tone lies, in an enharmonic series, between tetrachords of the form *q q d*, and not between those of the form *q d q* or *d q q*. We can use assumption **A** to guarantee that it lies between fixed notes, but that implies nothing about the order of intervals inside a tetrachord. Principle **P** or, with some qualifications, rule **R** will show that where there is a πυκνόν in the series, it must lie above, not below the disjunctive tone, at the bottom of the tetrachord. Hence *d q q*, *t*, *d q q*, at least, is impossible. But we had hoped to eliminate **P** and **R** in favour of something more fundamental: it is clear that this job has not yet been done.

In any case, the argument poses a substantial interpretative conundrum. The general line of thought seems plain enough: if we consider a series of conjunct enharmonic tetrachords of the form *q q d*, *q q d*, it is true that the upper and lower boundaries of each tetrachord are the lowest notes of πυκνά. If we now conceive the disjunctive tone as inserted between the boundaries of tetrachords of these sorts, its boundaries will be the boundaries of tetrachords, and the proposition to be proved apparently follows. But it is only too easy to dispute this reasoning. When the tone has been inserted, its upper boundary, certainly, will be the lowest note of a πυκνόν. But what of its lower boundary? In the sequence containing the additional tone, *q q d*, *t*, *q q d*, no πυκνόν appears in the space immediately above the first ditone: the upper boundary of the lower tetrachord is now apparently *not* the lower boundary of a πυκνόν, and neither, therefore, is the lower boundary of the tone.

The lower boundary of the tone, in fact, cannot possibly be the lower boundary of a πυκνόν that actually occurs in the sequence of which the tone is a part, and it is beyond belief that Aristoxenus should have supposed that it can, let alone that it must. His proposition must therefore be taken in some other sense, perhaps roughly that the note at the tone's lower boundary must be such that it *could* be followed by a πυκνόν above, in some sense of 'could', even though in the series containing the tone it is not. This notion of a note's potentiality is one that I shall explore later, when more informa-

tion about its use has been collected. Like **D** above, this proposition about the tone is important to the theorems that follow: let us call it **T**.

63.34–64.10 Two ditones cannot be placed in succession (ἑξῆς).

This proposition follows directly and obviously from **L**. It is therefore most curious that Aristoxenus makes no appeal to **L** here, offering instead an argument of a distinctly puzzling kind.

The argument depends on **D**, according to which the upper and lower boundaries of a ditone constitute, respectively, the lower and upper boundaries of πυκνά. It is true that from **D** the present proposition will apparently follow, since it seems clear that if there is a πυκνόν immediately above and below a ditone, there cannot be another ditone adjacent to the first. But again, this is not how Aristoxenus argues.

He reasons as follows. Suppose that one ditone is placed next to another. By **D**, there must be a πυκνόν immediately below the upper ditone, and similarly there must be a πυκνόν immediately above the lower ditone. Since *ex hypothesi* these two πυκνά stretch upwards and downwards from a common note, that between the two adjacent ditones, they will themselves be successive. But there cannot be two πυκνά in succession (from 62.34–63.5): hence there cannot be two successive ditones either.

The argument raises problems similar to those involved in the proof of **T**. In the hypothetical sequence *d d*, no πυκνόν actually appears. Aristoxenus' claim that by **D** πυκνά must exist above and below the note between the ditones cannot mean that the ditones in the sequence must actually be divided into smaller intervals and read as *3t/2 q q*, *q q 3t/2*, since the present group of propositions deals only with intervals that are melodically incomposite. Successive composite ditones, as such, are in any case perfectly permissible: in the conjunct chromatic series *s s 3t/2*, *s s 3t/2*, for instance, the four highest intervals, taken in pairs, constitute just such a sequence.[4] The sense of the argument must therefore be that though the sequence *d d* contains no πυκνά it in some sense implies them: just as the lower boundary of the tone is 'potentially' the lower boundary of a πυκνόν, so the note between the ditones is potentially or implicitly, but not actually, a note between adjacent πυκνά.

64.11–65.2 In the enharmonic and chromatic, two tones cannot be placed in succession.

Aristoxenus' argument presupposes both the sizes of the intervals in the tetrachords of the various forms of scale involved here, and the order in which they occur, designating them by reference to the notes that bound them: he proceeds, in effect, by an exhaustive survey of cases. He assumes also that the hypothetical extra tone, as well as the one that it is supposed to follow, will stand outside any tetrachord, an assumption grounded in the fact that no tone can in practice occur in tetrachords of the genera in question. Thus he argues, for instance, that if the λιχανός (the note below the highest interval in a tetrachord between fixed notes) is in the position appropriate to the enharmonic (that is, a ditone below the tetrachord's upper boundary), it will stand at a distance of four tones from the upper boundary of the additional tone. The sequence will be *d t t*, and this is plainly incompatible with **L**.

It is worth remarking that Aristoxenus had no need to proceed by this method. The enharmonic and chromatic genera share a feature distinguishing them from the diatonic, in that their tetrachords always contain a πυκνόν, whereas those of diatonic do not. The proposition could therefore be stated quite generally as one about systems

whose tetrachords contain πυκνά, and could be proved without reference to any particular tetrachordal divisions or named notes, since a tetrachord containing a πυκνόν cannot also contain a tone. Hence the extra tone must be placed next to the tone of disjunction, in order to form a sequence of two tones, and such an arrangement will always breach **L**. The residue of a fourth after a πυκνόν is always at least *3t/2*, and whether this residue is placed above the sequence of two tones or below it, the conditions laid down by **L** cannot be satisfied. Here, then, an argument involving reference to the order and sizes of intervals in particular generic sequences, and to named notes, could be replaced by one that is purely quantitative: it is interesting that Aristoxenus has preferred the former approach.

65.3–7 In the diatonic, a maximum of three tones can be placed in succession.

That a fourth successive tone is melodically impossible follows from **L**, and here it is **L** that Aristoxenus uses. He offers no argument to show that three successive tones in diatonic are possible, presumably because the fact is plain from the structure of the 'normal' diatonic sequence, taken in disjunction, *s t t*, *t*, *s t t*. It would, however, have been possible at least to prove its consistency with **L** for systems whose tetrachords contain no πυκνόν.

65.8–19 In the diatonic, two semitones cannot be placed in succession.

Suppose, first, that a second semitone is placed below the existing (ὑπάρχοντος) semitone. If we assume, though Aristoxenus does not make the assumption explicit, that the remainder of the series is not disturbed, this will give the sequence *s*, *s t t*, and the lowest note is in breach of **L**. Suppose that the new semitone is placed above the existing one: the resulting sequence, we are told, will then be chromatic and not diatonic. (The new semitone, in this position, will be part of the tetrachord to which the existing semitone belongs, and a complete tetrachord beginning *s s* must continue by *3t/2*.) The argument again presupposes the divisions of the tetrachords in the diatonic and chromatic genera, as well as **L**: again it could be generalised to show that semitones cannot occur successively in systems containing no πυκνά. (The proposition in this form is in fact trivial, since *s s* constitutes a πυκνόν.)

65.19–24 Aristoxenus announces that propositions about successions of equal incomposite intervals have now been completed. He has dealt with such successions in the cases of the ditone, the tone, the semitone, and the πυκνόν both in whole and in part. He has not explicitly mentioned cases of two other sorts: the complement of the πυκνόν in the fourth, when this interval is not a ditone, and intervals other than tones and semitones which occur in diatonic χρόαι other than the 'sharp' diatonic. (He mentions specifically a soft [μαλακόν] diatonic at 51.24–8, the intervals of whose tetrachords follow the pattern *s 3t/4 5t/4*.) The first of these cases, the alternatives to the ditone in the chromatic and possibly also the enharmonic (see, e.g., 49.7–19) is easily dealt with, since the arguments about the ditone will apply to them equally as a class defined in quantitative terms by reference to the fourth and the generalised concept of the πυκνόν. The variant intervals in the diatonic cannot be disposed of in this way, and need individual treatment. (Neither the arguments about the tone nor those about the semitone can be applied directly to them as a class.) This opens up the possibility that an indefinite number of other quantitatively defined intervals would also need individual discussion, given that tetrachords may in principle, as

Aristoxenus says from time to time, be divided in an indefinite number of different ways (e.g., 26.13–27). This is perhaps part of what he means when he says that a purely quantitative treatment of intervals will lead the science of harmonics into ἀπειρία: it will generate an indefinitely large number of distinct propositions, with no means of unifying them into coherent and comprehensible groups. (See 68.13–69.28, discussed in section 6 below.)

We move on, then, to consider sequences of unequal intervals.

65.25–30 A πυκνόν cannot be placed *both* above *and* below a ditone.
This is argued directly from **D** (63.5–20) and raises no immediate difficulties.

65.31–66.8 A tone may be placed above a ditone but not below it.
The problems involved in the proof of this proposition are ones we have encountered before. It is argued from **D** and **T**. By **D**, the lower of the notes bounding a ditone is the highest of a πυκνόν, and by **T** both the notes bounding a (disjunctive) tone are the lowest notes of πυκνά. Then in the sequence *t d*, the note between the intervals must have a πυκνόν both below and above, implying a sequence of two πυκνά, which is not permissible. Once again, the difficulty is that the πυκνά implied by the sequence do not actually appear in it: the notion of their implicit or potential presence has not yet been elucidated.

66.9–17 A tone may be placed below a πυκνόν but not above it.
This is argued from **T**, according to which both bounding notes of the tone are the lowest notes of πυκνά, on lines that are by now familiar. To place a tone above a πυκνόν is to generate an implied sequence of two πυκνά, and such a sequence is ἐκμελές. I shall continue to postpone discussion of the nature of this 'implication'.

66.18–22 In the diatonic, semitones cannot be placed both above a given tone and below it.
This is said to follow from **L**, and with certain qualifications, so it does. It will follow at once if it is assumed that the only incomposite intervals available in diatonic are the semitone and the tone. Even if this is not to be taken for granted, a route to the conclusion can still be found. The sequence *s t s* must be capable of being incorporated into a set of tetrachords. Then either the tone is a part of a tetrachord or it is not. If it is, the tetrachord must be completed by another tone, placed below the lower semitone or above the higher, and **L** is breached. If it is not, the tetrachord must contain two semitones and its remaining interval will be *3t/2*: such a tetrachord is chromatic, not diatonic. Further, if the tone is outside the tetrachords, it must be the tone of disjunction: the disjoined tetrachords must both be of the form *s 3t/2 s*, and two tetrachords each with a semitone at top and bottom cannot be disjoined by a tone. This will follow from Aristoxenus' pervasive use of **T**, together with the fact that in the conjunct series *s 3t/2 s*, *s 3t/2 s*, two semitones come together to form a πυκνόν. If a tone is inserted between the tetrachords, because its lower boundary is deemed to 'be', in some sense, the lower boundary of a πυκνόν, a πυκνόν and a part of a πυκνόν will stand in succession, and this is not allowed.

66.22–25 A semitone may be placed both above and below a sequence of two tones or of three tones.
The argument is merely that **L** is not breached, which is interesting, since Aristo-

xenus typically treats conformity to **L** as a necessary but not sufficient condition of melodic propriety (53.32–54.18). This treatment is not unrelated to the fact that the concept 'tone' is here used purely quantitatively, no attempt being made to distinguish tones inside tetrachords from tones of disjunction. Though *s t t s* and *s t t t s* are legitimate segments of the diatonic series, Aristoxenus would insist that *s t t s* can involve no disjunction (but is a fragment of the conjunct sequence *s t t*, *s t t*), while only the third tone in *s t t t s* can be disjunctive (*s t t*, *t*, *s t t*, but not *s t*, *t*, *t s t* or *s*, *t*, *t t s*). **L** by itself cannot determine the position of the disjunctions in a diatonic sequence: Aristoxenus' formal treatment of the matter (68.2–12) necessarily introduces non-quantitative assumptions, one of which, as we have already seen, will have to be **A**.

V 'Potential' sequences from the tone and the ditone

Here it will be convenient to pause and consider more fully Aristoxenus' use of **D** and **T**. **D** states that a ditone must be succeeded by a πυκνόν both above and below, which is true in a direct sense only in the conjunct series. **T** states that a (disjunctive) tone is bounded by notes both of which are the lowest notes of πυκνά: this is directly true only of its upper boundary (and, of course, only in sequences containing πυκνά, i.e., not in diatonic sequences).

Of the several arguments in which **D** and **T** have appeared as premises, that given at 65.32–66.8 may be taken as an example. A tone cannot be placed below a ditone, because their common boundary must be the highest note of a πυκνόν (by **D**), and the lowest note of a πυκνόν (by **T**). Hence two πυκνά will be placed in succession, and this is melodically impossible.

We have seen that Aristoxenus cannot mean that these πυκνά will actually appear in the sequence *t d*. They are somehow implied, and the implication involved is such that it requires the two πυκνά to be treated as parts of the same series. If the series containing them is melodically improper, (ἐκμελές) so is the sequence that implied this series.

One way of construing this requirement is to say that all the intervals involved, whether actual or implied, must be mapped onto a single series, and that it is this series whose legitimacy is tested. The actual series is *t d*: the implied series consists of πυκνά running in each direction from the note common to the tone and the ditone. In the combined series the πυκνά will therefore subdivide parts of both the tone and the ditone, giving *s q q*, *q q 3t/2*. Plainly this series is intolerable. Yet this interpretation cannot be allowed, for two reasons at least.

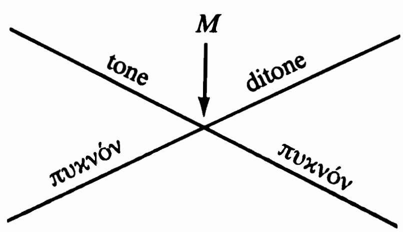
First, as we have seen, the rules about tones and ditones apply only where they are melodically incomposite: a theorist who proposed that an incomposite tone may lie below an incomposite ditone could hardly be understood as thereby proposing the combined series set out above.

Secondly, and more straightforwardly, this interpretation would also make illegitimate the sequences implied by Aristoxenus' own proposition **T**. Both bounding notes of the tone are the lowest notes of πυκνά: hence, e.g., the series *q q d*, *t*, *q q d* 'implies' the sequence *q q d q q*. If we put these together in the way suggested above, we get *q q d*, *q q s*, *q q d*, which breaks Aristoxenus' rules of succession (in particular, **L**) as flagrantly as anything could.

There is, however, a more promising way of interpreting the requirement that the 'implied' intervals must be capable of standing as part of a single series. In criticisms of

his predecessors' analyses of octave-systems or ἁρμονίαι, and particularly those of the school of Eratocles, Aristoxenus complains that, though they recognise that after a sequence of intervals spanning a fourth the melodic series 'splits in two', they do not say whether this may happen after just any fourth, or only after certain special ones (5.9–22). It seems clear that the issue is that of the location of the disjunctive tone, the assumption being that where a tetrachord is followed by another in disjunction, it would always have been equally legitimate to proceed by conjunction instead. Hence where disjunctive tones occur, the series splits into a pair of alternatives, as indeed it does at the one point in the standardly accepted two-octave system where tetrachords may be disjoined. (On reaching the note μέση, one may proceed upwards to a tetrachord in disjunction, the tetrachord διεζευγμένων, or to one in conjunction, the tetrachord συνημμένων: see section 1.) The question that Eratocles neglected, then, is whether disjunction can occur as an alternative to conjunction between tetrachords of just any form, at any point in the series, or only between tetrachords of one particular shape. It is Aristoxenus' contention, of course, that the bifurcation of the series can occur only between tetrachords bounded by fixed notes, and that the tetrachords in question must, in the enharmonic, be those of the form *q q d*, not *q d q* or *d q q*, or in the diatonic, *s t t*, not *t t s* or *t s t*, and so on. Eratocles may well have agreed with Aristoxenus' reading of the facts: what he failed to provide was a method of demonstrating that these things are so, and why. The propositions in book 3 that concern the tone are mainly to be construed as attempts to repair this omission: those involving **T**, which relates specifically to the *disjunctive* tone, are plainly cases in point.

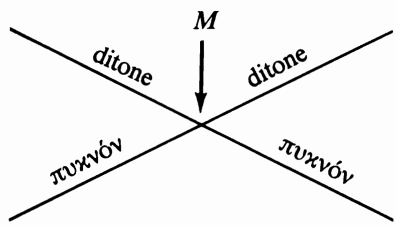
Thus, it may be that the πυκνά implied by the sequence *t d* are to be understood as belonging to an *alternative* series whose possiblility is implicit in that of *t d*. Let the note common to the tone and the ditone be *M*. Then from *M* it is possible, *ex hypothesi*, to proceed upwards by a ditone and downwards by a tone: according to the rules that **D** and **T** incorporate, it is also possible, as alternative steps, to proceed downwards or upwards by a πυκνόν. The structure can then be represented as follows.



If we then assume that by whatever route we reach the common note from below, it is permissible to proceed by either of the alternative upward paths, we may arrive at *M* through a tone and leave by a ditone, arrive through a tone and leave by a πυκνόν, arrive through a πυκνόν and leave by a ditone, or arrive through a πυκνόν and leave by a πυκνόν. Since this last sequence is not permissible, neither is the whole structure of alternatives to which it belongs, and hence neither is the sequence *t d*, out of whose implications the set of alternatives was generated.

This interpretation is attractive, and I believe it to be broadly correct. But it leaves two problems, one rather general, the other specific. Let us take the latter first. The

notion that where tetrachords are disjoined by a tone it should be equally possible to proceed to a tetrachord in conjunction, which produces the bifurcation of the scale and the sets of alternative continuations, seems to fit Aristoxenus' conception of the role of this tone very adequately, and it is appropriate in all cases where he makes use of **T**. On one occasion, however, **T** is not involved, and an argument parallel to the one we have considered is based on **D** alone. This occurs in the proof of the proposition that one ditone cannot follow another (63.34–64.10). It cannot, because their common note, *M*, will be the lowest note of a πυκνόν in virtue of being the upper boundary of a ditone, and will be the highest note of a πυκνόν in virtue of being the lower boundary of a ditone. If we interpret this as before, the structure generated by the sequence of ditones will be this.



But on what grounds could Aristoxenus insist that the sequence of ditones implies the bifurcation of the series, and the possibility of alternative continuations? The question where a disjunctive tone can lie can properly be conceived as the question where the scalar series can divide, the question that Eratocles neglected to consider. But there seems no reason to suppose that an innovator who proposed the legitimacy of the sequence *d d* must thereby imply the existence of a 'disjunctive ditone', and another form of scalar bifurcation. There is no evidence and no likelihood that such a theoretical amphibian was ever even imagined. Yet if the hypothetical extra ditone need not be conceived as one of a pair of parallel alternatives, the argument we have constructed on Aristoxenus' behalf cannot apply.

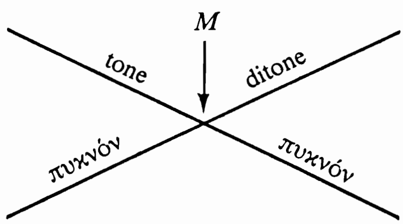
Let us turn to the more general problem. The evidence on which **D** and **T** are based is drawn from facts about particular forms of the scalar series. **D** is argued from the behaviour of tetrachords in conjunction, **T** partly from their behaviour in disjunction, partly from their behaviour in conjunction. That is, the clause of **T** which asserts that the tone's upper boundary is the lowest note of a πυκνόν is directly true for the disjunctive series, while the same assertion about its lower boundary is not directly true for any series in which this tone actually occurs, but depends on the role of this boundary as the highest note of a tetrachord. When the tone is not present, but only then (i.e., when the tetrachords are in conjunction), the highest note of a tetrachord is the lowest note of a πυκνόν. Now Aristoxenus' uses of **D** and **T** require their application to cases other than those in which they were originally found to hold. This suggests a form of induction, but it is an odder form than usual: one does not have to embrace a Popperian or even a Humean position to find it suspect.

Aristoxenus has found, for example, that one well established (though in his time almost outmoded) form of scale contains a ditone with two quarter-tones (the enharmonic πυκνόν) below it in every tetrachord between fixed notes. On what principle does it follow that every legitimate form of scale containing a ditone *must* therefore

place a πυκνόν below it? Even allowing that legitimate scales are subject to the constraints of **L** and **A**, this thesis does not follow: neither puts any constraints on the order in which intervals occur inside the tetrachord. If we presuppose **P** in addition, the thesis will follow, but **P** is evidently no more than an inductive generalisation from known cases: cases conflicting with **P** could readily be generated in practice, and it is not obvious that **P** could then be used to undermine their status. If **P** is merely an inductive generalisation, these cases could equally well be treated as counter-examples that prove **P** false.

The difficulty reaches to the heart of Aristoxenus' enterprise. Greek harmonics is patently to some extent a normative science. It cannot hope to show that certain sequences are physically impossible (a project of a kind in which induction has often been thought to play a part), only that they are melodically unacceptable. It is very far from clear how the fact that a certain sequence has not hitherto been used can be exploited to show that it ought never to be used: must this alleged science rest on nothing but conservative prejudice?

The proposition that the ditone also has a πυκνόν above it, and **T**'s thesis that the tone's lower boundary is the lowest note of a πυκνόν, have oddities of a different sort, as we have seen. The πυκνόν above the ditone does not appear as such in the disjunct series, and that above the lower note of the tone never appears when the tone does. We have found that it is possible, with some qualifications, to interpret the 'potential' or 'implied' presence of these πυκνά as related to the possibility of alternative continuations from a given note. But from the fact that when this note is approached in a specific way, the series may legitimately continue by a πυκνόν, for example, why does it follow that continuation by a πυκνόν must be possible no matter by what interval the note is reached? Consider again the first of the two diagrams given above.



Why should we not merely conclude, from an application of Aristoxenus' laws to this structure, that if *M* is approached through the lower πυκνόν, the series must continue with a ditone and not with the second πυκνόν? Or why, more radically, should the fact that in some scales a tone has a πυκνόν above it entail that this πυκνόν must be a legitimate alternative to the ditone, in the hypothetical scale in which the sequence *t d* occurs? It seems that some kind of inductive inference is being used here not merely to rule out what has not been done before, but to insist that what *has* been done before must be a legitimate option in any scale whatever.

I think that Aristoxenus' procedure can in fact be defended, and answers found to the questions we have raised. These answers will help to resolve difficulties that we have found elsewhere. A survey of the remaining arguments of book 3, together with some general remarks inserted by Aristoxenus along the way, will help us to sharpen the necessary tools.

VI Routes of progression

Most of the propositions in the rest of the book concern the number of routes (ὁδοί) that can legitimately be followed in either direction from specified intervals, interval-sequences, and notes. They add little that is substantially new, and are supported by reasoning intended, for the most part, merely to summarise arguments already given. They constitute a reorganisation of existing conclusions, rather than breaking fresh ground. Certain features of the ways in which they are presented, however, will give valuable help in the interpretation of their predecessors.

66.27–67.25 After a brief clause concerning routes from the semitone, excised, probably correctly, by Macran and da Rios, Aristoxenus proceeds to show how many routes there are, first from a ditone, and next from an (enharmonic) πυκνόν. From a ditone there are two routes or possible continuations upwards and only one downwards: one may move upwards to either a tone or a πυκνόν, but to no other interval; and downwards one may move only to a πυκνόν. From a πυκνόν there are two routes downwards and one upwards, down to a tone or a ditone, up to a ditone and to nothing else. These conclusions rest squarely on propositions already proved, and call for no further comment.

67.25–68.1 Similar propositions are next offered concerning the tone (of disjunction), still in the context of the enharmonic genus. It follows from previous conclusions that there is only one ὁδός in each direction from this tone, upwards to a πυκνόν, downwards to a ditone.

68.1–12 More interesting features begin to appear as soon as the focus shifts from the enharmonic to the chromatic and diatonic genera. The proposition about the routes from the tone in the enharmonic, Aristoxenus says, applies equally to chromatic sequences, except that the ditone is replaced by 'the interval between the μέση and λιχανός', and the πυκνόν, instead of being a pair of quarter-tones, will be whatever size it is in any given shade (χρόα) of the genus. It applies also to diatonic sequences, in that there is just one route in each direction from 'the tone common to the genera', downwards to the interval between the μέση and λιχανός, whatever size it may be in a given diatonic χρόα, and upwards to the interval between the παραμέση and τρίτη (the lowest and second-lowest notes of the tetrachord διεζευγμένων).

Two points stand out. One is that the routes from the tone are no longer specified quantitatively as involving intervals of particular sizes, but are described by reference to the notes that bound them. Secondly, in the application of the proposition to the diatonic genus, the tone itself has to be identified not merely by its size, but as 'the tone common to the genera'. This, or some similar formulation, is plainly necessary, since the interval of a tone can appear in any of three roles in diatonic sequences, as the middle or the upper interval of a tetrachord, or as the interval of disjunction, and it is only the last of these that Aristoxenus wishes to consider. The identification of the relevant tone depends on an assumed framework of conjunct and disjunct tetrachords that does not alter with change of genus. The specification of intervals by reference to the names of their bounding notes involves still more detailed and complex assumptions about the standard note-series as a whole. It makes no sense to call an interval 'the one between the μέση and λιχανός, unless these names are attached to notes that are in some

sense determinate and identifiable. The μέση, of course, is a fixed note, identifiable by its position in the tetrachordal framework, in terms that need only refer to quantitative relations between it and the other elements of the framework. The λιχανός is not: its position relative to the fixed boundaries of the tetrachords is variable through an indefinite number of locations, though within determinate limits. It must therefore be identified in another way.

Aristoxenus is perfectly well aware of what he is doing. He remarks that some people find the present proposition puzzling, and answers their difficulties in a grand methodological digression that runs from 68.13 to 69.28. People find it hard to understand, he says, why instead of saying that there is one progression in each direction from the tone, we should not say that these routes are indefinite or infinite (ἄπειροι) in number. After all, the sizes of the interval between the μέση and λιχανός are ἄπειρα, and so are those of the πυκνόν. Aristoxenus' reply begins by pointing out that if this is said of the progressions from the tone, something similar can equally be said of those from the other intervals previously discussed, the πυκνόν and the ditone. The routes from the tone are limited to one in each direction in precisely the same sense in which the routes from the others are limited.

At this point the reader will naturally ask how we are to understand this 'sense' in which they are limited. Aristoxenus answers first with the assertion that the routes are to be considered in each χρόα of each genus separately. This will of course yield the result he wants, but taken by itself it is mere stipulation. Why should the χρόαι be taken one at a time? Aristoxenus explains further. Whatever musical item is under consideration, we must specify it and organise it into a scheme of scientific knowledge (τιθέναι τε καὶ τάττειν εἰς τὰς ἐπιστήμας) in the respect in which it is determinate (καθ' ὃ πεπέρασται), and if it is indeterminate we must leave it alone (εἰ δ' ἄπειρόν ἐστιν ἐᾶν). Now things to do with melody seem to be indeterminate, to some degree (ἄπειρά πως φαίνεται εἶναι), in respect of the sizes of the intervals and the pitches of the notes: they are determinate and orderly (πεπερασμένα τε καὶ τεταγμένα) in respect of δυνάμεις, εἴδη, and θέσεις. Thus in their δύναμις and their εἴδη, the downward routes from the πυκνόν are limited (ὡρισμέναι), and two in number, since the one that proceeds by a tone leads the εἶδος of the σύστημα into disjunction, while that which proceeds by the alternative interval, no matter what its size, leads it into conjunction. Plainly, then, Aristoxenus continues, there is just one route in each direction from the tone, and the two routes taken together are causes of just one form of system (ἕνος εἴδους συστήματος αἰτίαι), that of disjunction. He concludes that both what he has said and the brute facts demonstrate that if one seeks to consider the routes from given intervals not in one χρόα of one genus at a time, but all together, one will inevitably collapse into indeterminacy (εἰς ἀπειρίαν ἐμπεσεῖται).

It is tempting, and might even be correct, to read this passage as a reminiscence of a well known passage of Plato's *Philebus* (16c–17e), endorsing some of what Plato says there but rejecting equally important aspects of his account. On the other hand, it may merely reflect Aristoxenus' independent familiarity with Pythagorean uses of the notions of πέρας and ἀπειρία, as well as with the quantitative form of harmonics characteristic of the school. He reaffirms, in either case, the idea that the province of science is the determinate, and also that musical phenomena are not determinate in all respects. But he disputes the Pythagorean and Platonist assumption that what is determinate about melodic progressions is something that can be fully described in quan-

titative terms by reference to the pitches of notes and the sizes of intervals. His disagreement is independent of whether intervals are to be quantified in his own way, as quasi-linear 'distances' between notes conceived as 'points', or as ratios between notes where the notes are treated as magnitudes (the method favoured by Plato and by all Pythagorean theorists). No matter how quantification is approached, the results of harmonic science cannot be quantitatively expressed, since the determinate facts about melodic progressions are not facts about successions of intervals of this and that size. They are facts about δυνάμεις, εἴδη and θέσεις: we shall consider more fully in section 7 what these conceptions amount to, and what the implications of Aristoxenus' standpoint are. At the least it must lead us to reappraise the purpose of the theorems. We have already put together an impressive collection of hints that they are not to be construed simply as arguments from quantitative premises to quantitative conclusions, and the present passage may help to provide a more coordinated view of the other considerations that lie in the background. Nevertheless, it is clear that quantitative rules and specifications have substantial parts to play: Aristoxenus is not entitled to dismiss them as glibly as he seems to here.

One feature of the argument in the digression needs to be tidied away before we pass on. The discussion of δύναμις is introduced as an explanation of why progressions from given intervals must be studied in one χρόα of one genus at a time, and it is not altogether clear how the argument works. If we consider one χρόα at a time, it will after all be possible to specify the progressions precisely and definitely in quantitative terms. But perhaps Aristoxenus' point is just that such precise *quantitative* description is only possible χρόα by χρόα, and that this is true because the determinate facts about forms of progression from a given interval in every variety of scale taken together are not quantitative facts. Nor is it any quantifiable features of the interval and the progression that makes them the *same* interval and the *same* progression in scales of different kinds. The truths that unify all such progressions, though not quantitative in form, ensure that in any given χρόα the progressions are quantitatively determinate, and that a χρόα-by-χρόα approach will therefore give determinate results. But an enumeration of quantified progressions for one χρόα after another will not by itself give an overall scientific understanding of the one kind of progression that they all exemplify: the enumeration can never be complete, and even if it could, no purely mathematical rule will put order into its ἀπειρία, explaining the manner in which the plurality of quantitative types constitutes a coherent and comprehensible 'one'.

We must now investigate the remaining propositions about routes or progressions (ὁδοί).

69.29–70.14 In the enharmonic and chromatic, every note is part of a πυκνόν.

This proposition is not directly concerned with ὁδοί: it serves to prepare for the theorems that follow. Its argument hangs on characteristic uses of **T** and **D** which I shall not now consider further. Its conclusion is problematic only in the sense that the highest note of a tetrachord disjoined from the one above it is not in a direct sense part of a πυκνόν, but is so only through the 'implications' of **T** and **D**.

70.15–20 There are three positions for notes in the πυκνόν.

By this Aristoxenus means simply that there is a lowest, a middle and an upper note in every πυκνόν, and no more. That there are no more follows, as he says, from

the proposition that no πυκνόν can be followed by another or by a part of one: this rules out, for example, the subdivision of the chromatic πυκνόν *s s* by the insertion of additional notes within its boundaries.

Aristoxenus now considers the ὁδοί that are available from each note of the πυκνόν in turn.

70.21–71.4 From the lowest note of the πυκνόν there are two ὁδοί in each direction, downwards to a tone or a ditone, upwards to a πυκνόν or a tone.

The first part of the proposition raises no obvious difficulties, since it follows readily from earlier conclusions, but one feature of Aristoxenus' argument is worth emphasising. It has previously been shown, he says, that from the πυκνόν there are two routes downwards, one to the tone and one to the ditone,[5] and to say this is the same as saying that there are two routes downwards from the lowest note of the πυκνόν, since it is by this that the πυκνόν is bounded. It seems a small point, but it is a significant one, as I shall argue later, that Aristoxenus finds it important to convert propositions about progressions from specified intervals into ones about progressions from specified notes.

The argument for the second part of the proposition, concerning the upwards routes, is altogether remarkable. One might have imagined that since the subject of discussion is *defined* as the lowest note of a πυκνόν, progression upwards from it must be to the πυκνόν and to nothing else. But it has been shown, Aristoxenus says, that from the ditone there are two routes upwards, one to a tone and one to a πυκνόν. To say this, however, is the same as saying that there are two routes upwards from the upper bounding note of the ditone. Since it has been shown that the upper bounding note of the ditone is the lower boundary of a πυκνόν, there must be two routes upwards from the lower boundary of a πυκνόν.

The reasoning makes striking use of tactics similar to those involved in earlier applications of **T** and **D**. In one of the progressions upwards from the lowest note of a πυκνόν, i.e., the progression to the tone, the πυκνόν whose lowest note it is does not appear. It is the lowest note of a πυκνόν, then, in some implicit or potential sense, just because it is the highest note of a ditone: we have found previously that Aristoxenus insists on calling this note the lowest note of a πυκνόν even when it is in fact succeeded by a tone. It begins to look as though the various characterisations that can be attached to a note are taken to belong to it not in virtue of the intervals actually surrounding it, but in virtue of something intrinsic to itself, which remains present in it even when it is not expressed in any actual progression of intervals. To mention the upper note of a ditone, or the lower note of a disjunctive tone, or the lowest note of a πυκνόν, is not simply to mention three different locations which in certain forms of scale may coincide: it is to mention three aspects of an entity which is always identically the same note, and whose nature or essence is constituted, at least in part, by the unification of these three aspects. In that case it becomes natural to ask whether it is not, after all, the intervals of a sequence that determine the legitimate specifications of its notes, but rather the independent characters of the notes that determine the sequences of intervals in which they are capable of appearing. I shall try to pursue this suggestion further in section 7.

71.5–22 From the highest note of a πυκνόν there is one ὁδός in each direction.

The argument looks straightforward, if a little cumbersome. It has been shown that from the πυκνόν there is only one route upwards, to a ditone. To say this is the same as saying that there is only one route upwards from the highest note of a πυκνόν. It has also been shown that there is only one route downwards from a ditone, that to a πυκνόν, and to say this is the same as saying that there is only one route downwards from the lower bounding note of the ditone. But since we also know that this note is the highest note of a πυκνόν (by **D**), this is also the same as saying that there is only one route downwards from the highest note of a πυκνόν.

There is nothing contentious about Aristoxenus' conclusion. But the apparently unnecessary complexities of his argument show him, once again, using conclusions about intervals to establish features of an identifiable note. To say that the ditone is followed by a πυκνόν below is to identify the ditone's lower boundary as that note whose nature it is to require a πυκνόν below, a ditone above. The idea that progressions are determined by the characteristics of notes is more obviously at work in cases where from a given note several different progressions are possible and are deemed to be present potentially even when they do not actually occur. But it lies equally, though less visibly, behind the reasoning of the present argument.

71.23–72.12 From the middle note of the πυκνόν there is one route in each direction.

The proposition sounds as if it were true by definition, yet Aristoxenus once again proceeds to prove it by a complex argument involving applications of **T** and **D**. The gist is that since each note bounding a tone or a ditone is thereby a boundary of a πυκνόν, and since from the middle note of a πυκνόν there is a part of a πυκνόν in each direction, to move to a tone or a ditone from this note would (implicitly) be to place a πυκνόν next to part of a πυκνόν, which is not permissible. This line of reasoning indicates that to call a note the note below a ditone is to define it as the highest note of a πυκνόν (even where that πυκνόν does not actually occur, as it would not in the case hypothetically envisaged), as surely as calling it the middle note of a πυκνόν defines it as having part of a πυκνόν on either side (even though it would not, if an incomposite ditone were placed immediately above it). The focus, once again, is on the defining characteristics or 'potentialities' of a note, not on the sequence of intervals that actually appears around a note on this or that occasion.

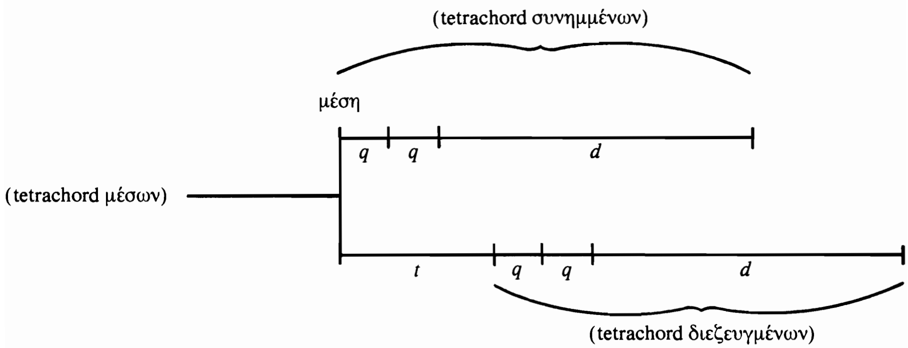
72.13–27 Two notes that differ in their position in the πυκνόν (κατὰ τὴν τοῦ πυκνοῦ μετοχήν) cannot with melodic propriety (ἐμμελῶς) be placed upon the same pitch (τάσις).

This proposition is perhaps intended as a summary of the conclusions of the preceding propositions: the lower bounding note of the ditone and the middle note of a πυκνόν, for instance, differ in their position in the πυκνόν, and it has been shown that they cannot coincide. The argument that Aristoxenus now offers is in effect a generalisation of the previous ones.

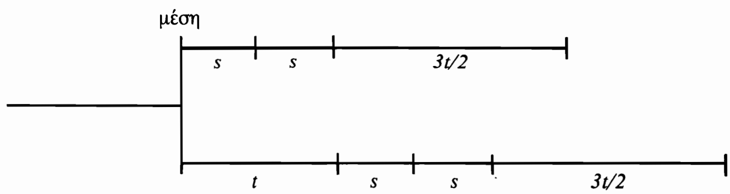
In his statement of the proposition, however, Aristoxenus seems to make one of his very rare slips, and the mistake is instructive. It is true, and readily proved from familiar resources, that no one note may have two roles in the πυκνόν: it cannot be both the middle note of a πυκνόν and the highest note of one. But this is not the same as saying that two notes differing in their position in the πυκνόν cannot stand at the same

pitch: it is not hard to find an example in which Aristoxenus would certainly agree that they do.

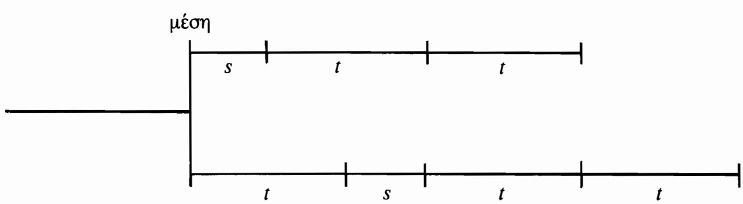
In any genus, the series of intervals bifurcates above the note μέση, proceeding by conjunction to the tetrachord συνημμένων, by disjunction to the tetrachord διεζευγμένων. In the enharmonic genus this double sequence can be represented as follows.



In the 'tonic' χρόα of chromatic, it is this:



In the 'sharp' χρόα of diatonic, it is this:



The rule that Aristoxenus has stated is not broken in the enharmonic, since in that genus no note of the one tetrachord falls on the same pitch as a note of the other. In the diatonic it is not breached directly, because diatonic sequences contain no πυκνά. But in the tonic chromatic the second note from the μέση in the tetrachord συνημμένων (the παρανήτη συνημμένων) falls on the same pitch as the first note from the μέση in the tetrachord διεζευγμένων (the παραμέση). The παραμέση is the lowest note of a πυκνόν, the παρανήτη the highest: hence the rule, in the form given, is broken.

The mistake is one that Aristoxenus ought not to have made, in view of his clear assertion (69.6–11) that it is not in respect of pitches (τάσεις) that melodic phenomena are determinate and law-abiding, but in respect of δυνάμεις, εἴδη and θέσεις. The pitches of the παραμέση and παρανήτη may be the same, but their θέσεις, their positions in their tetrachords and in the whole system, are not, and no more are the εἴδη, the overall structures, of the sequences in which they occur. The concept of δύναμις will be considered further in the next section, but we may anticipate that discussion to the extent of saying that it is most prominently conceived as a note's capacity to determine which intervals may lie on either side of it. As such it is the basis of both εἶδος and θέσις, or at least intricately connected with them; and plainly the δυνάμεις of the παραμέση and παρανήτη differ. It should also already be apparent, though this point too will be reviewed again below, that a (named) note is to be identified by reference to its δύναμις, not to its pitch. Then there is all the difference in the world between saying that no one note can have two δυνάμεις, incompatible with one another (displayed, for example, by its holding two different positions in the πυκνόν), and saying that no two notes, incompatible in these respects, can fall on the same pitch. They can and do, not only in the case of the tonic chromatic, but also in diatonic, where notes that differ in this way coincide at two points in the double series set out above.

Aristoxenus' formulation cannot be rescued by the stipulation that the conjunct and disjunct sequences, like the different χρόαι, should be considered one at a time. His own procedure shows that notes are to be defined by reference to their roles in both sequences simultaneously: otherwise **T** is false, and neither **T** nor **D** can be applied in the ways we have been studying.

The remaining propositions of the book, which is incomplete, are of less importance for present purposes. 72.28–74.8 explains how many different incomposite melodic intervals there are in each of the genera. 74.9–16 gives a definition of difference of εἶδος or σχῆμα, which is helpful in interpreting his uses of the notion elsewhere. Such difference occurs when 'in the same magnitude (of interval) constituted out of the same incomposite intervals, the order of the incomposite intervals is altered'.

The book ends (74.17–25) with the proposition that there are three such (εἴδη) of the fourth. One has the πυκνόν at the bottom, one contains a ditone with a δίεσις (quarter-tone) on either side, and one has the πυκνόν above the ditone. These εἴδη are of course not all possible as forms of tetrachords between fixed notes, which have previously been Aristoxenus' major concern: they arise when one asks what εἴδη, or patterns of internal structure, an interval of a fourth in a given genus can have regardless of its position in the system or the characters of its bounding notes. There can be little doubt that Aristoxenus would have gone on to enunciate a similar proposition relating the fifth, and to combine these results in an analysis of the forms of the octave. He plainly believed this latter task to be of central importance, and criticises his predecessors for failing, in their accounts of the octave-species or ἁρμονίαι, to start from an analysis of the octave's principal components, the fourth and the fifth, and of the forms of σύνθεσις in which they can be put together. Only through such a procedure can it be proved that certain of the logically possible species of the octave are melodically permissible, while others are not. (See especially 6.19–31.) Now that the

legitimate sequences of incomposite intervals have been enumerated, and the basis of their legitimacy and the impropriety of others explained, it has become possible to give such analyses of the fourth and the fifth, and to put together out of them, together with the rules of σύνθεσις (particularly those concerning conjunction and disjunction), a scientifically rigorous and well-founded enumeration of the forms of the octave.

VII The concept of δύναμις and the roots of explanation

We have found that a purely quantitative interpretation of Aristoxenus' theorems cannot be sustained. Though it is certainly part of his project to establish the sequences in which intervals of given sizes can occur, and though some of his premises are quantitative in form, his arguments also presuppose principles of a different order, and by no means all his conclusions are quantitatively expressed. Correspondingly, if intervals are conceived as 'distances' of specified dimensions, so that two intervals are the same if their dimensions are the same irrespective of where they stand in the structure of the system, these intervals cannot be the primary reference-points of harmonic science. They cannot be the entities whose patterns of behaviour the underlying principles and their subordinate propositions describe. Aristoxenus says as much, and his procedure reflects it. What is being analysed is something of whose behaviour the regular interrelations of quantifiable intervals are an aspect, but whose properties cannot be reduced to the sum of such interrelations.

It has been necessary to make use of additional assumptions or principles that seem to fall into two main types: but their important implications converge. In the first place, Aristoxenus evidently takes for granted something that corresponds in outline to our assumption **A**: the framework of fixed notes bounding tetrachords in conjunction and disjunction is given, not derived. On occasion, and frequently in the later theorems, this assumption is supplemented by irreducible references by name to those notes of the system which are not fixed, and intervals are identified by the positions in the system that their bounding notes are defined as occupying. Such positions cannot be pinned to a single locus by quantitative coordinates: a note is the same note in several occurrences not in virtue of retaining the same pitch relative to other notes, but by filling the same functional or dynamic niche in the overall structure. It is neither a necessary nor a sufficient condition of *N*'s being the same note as *M* that they should stand in the same pitch-relations to other given notes. We therefore need a fuller account of what it is to be some one identifiable note, of how the structures containing such notes are understood, and of how propositions concerning them are related to propositions about the sequences in which intervals, quantitatively conceived, may and may not occur.

Secondly, we have seen that if the theorems are understood as concerned primarily with interval-sequences, there are very serious difficulties in certain pervasive forms of inference or argumentative procedure. These are most striking in Aristoxenus' uses of **T** and **D**, though they are not confined to them. If we treat it as a proposition about actual successions of intervals, **T**, for example, is simply false: there is no πυκνόν lying immediately above the lower boundary of any tone. I suggested that some of the relevant arguments may be reconstructed round the notion of alternative sequences: where a sequence proceeds upwards from a tetrachord to a disjunctive tone, for instance, it could have proceeded instead to the lower intervals of a tetra-

chord in conjunction with the first. But in some cases this interpretation seemed much less appropriate, and in general we found no convincing grounds for the thesis that such alternatives *must* be implicit in sequences of these sorts. Granted that in some accepted scale-forms a πυκνόν lies above a ditone, why does it follow that in *every* scale-form containing a ditone it is *always* legitimate to proceed upwards to a πυκνόν? What ensures that this πυκνόν is always potentially present? And what requires us to accept that if a tone 'implies' the existence of a πυκνόν above, and a ditone that of a πυκνόν below, then the hypothetical sequence *t d* implies the existence of two πυκνά in sequence, one running upwards and one downwards from the common note? If the πυκνά actually occur, neither the tone nor the ditone can do so: how can the sequence *t d* imply the sequence of πυκνά, if the latter can only occur when the former does not, and is therefore not present to exercise its powers of implication?

In some of the later theorems that incorporate these forms of reasoning, a promising line of interpretation seemed to be one that treated the propositions as concerned, primarily, not with the ways in which one interval can follow another, but with the intervals capable of lying between specified notes, the notes themselves being identified only partly by the sizes of the intervals they delimit in the series under consideration. If their identity depended wholly on these sizes, the proposition that there is only one route in each direction from the middle note of a πυκνόν, for instance, would be true by definition, whereas Aristoxenus treats it as one that could coherently, though mistakenly, be supposed false: the fact that it is true calls for a complex demonstration from independent principles. Hence the specification of *M* as the middle note of a πυκνόν does not by itself entail that *M* always, necessarily, occurs as part of a sequence in which it has one interval of the πυκνόν on either side (though in fact it does): it is not absurd *a priori* to suggest that it might on occasion be, for example, the lower boundary of an incomposite ditone. The phrase 'the middle note of a πυκνόν' identifies something that would remain the same note, even if the sizes of the intervals surrounding it changed. Perhaps, then, it is notes, not intervals, that carry 'implications' about succeeding intervals. It is thoroughly obscure how a πυκνόν could be implied by a tone in a sequence where that tone is not actually present. But if the notes that bound the tone (in those sequences where it does occur) retain their identity whether the tone is present or not, their powers of implication, whatever these may be, can intelligibly be conceived as remaining intact, unaffected by changes in the sizes of the intervals among which they appear on this occasion or that.

Here again, then, it becomes important to try to unravel the notion of the identity and character of a note. Both kinds of difficulty point unmistakably to the hypothesis that the whole collection of theorems is best understood as a systematic exposition of the ways in which such characters are interrelated. In that case the rules about interval-sequences will be secondary, derived from and explained by truths about the natures of notes. The laws of harmonic science, which express these truths, will be discovered by analysis of the structures implicit in the system as a whole, whose organisation is determined by the interaction of the potentialities or δυνάμεις constituting the notes which are its elements.

We must turn, then, to what Aristoxenus himself says about the laws of harmonic science, and especially about the concept of δύναμις. Nowhere in the parts of his work that survive does he define or deliberately explicate this concept, though it was plainly his intention to do so. He tells us (36.2–14) that after a study of the genera and

of intervals (διαστήματα), the third part of harmonics concerns φθόγγοι (notes) since, he says, intervals by themselves are not sufficient (αὐτάρκη) to give understanding of notes. The reason is that virtually every size of interval is 'common to several δυνάμεις'. (The point seems to be that movement through the interval of a fourth downwards, for example, is possible from any of many different notes: this possibility of moving an interval of a given size is thus an element in each of many different δυνάμεις, and so cannot constitute the whole essence of any of them.) The study of musical notes, Aristoxenus continues, requires saying how many there are, by what they are recognised or identified (γνωρίζονται), and whether they are τάσεις (pitches) as most people suppose or δυνάμεις: and an account must be given of what δύναμις itself is.

His phraseology here, as well as his dismissive remarks about pitches elsewhere (e.g., 69.6–8), makes it clear that in his view a note is a δύναμις, not a pitch. The promised account of δύναμις, however, is lost, if he ever formulated it. Since a note is a δύναμις, the question how notes are to be identified and how many there are, concerns the identification and enumeration of δυνάμεις. The fact that each named note is to be treated as instantiating a δύναμις is further indicated at 34.1–5, a passage that also helps in interpreting the claim at 36.2–14 that we cannot understand notes by studying intervals alone. Sometimes, Aristoxenus says, while the size of interval remains the same we designate it differently, calling it in one occurrence the interval between the ὑπάτη and μέση, for instance, and in another that between the παραμέση and νήτη: for it can happen that while the interval's size remains constant, the δυνάμεις of the notes change. To call a note παραμέση, then, is to attribute to it a definite δύναμις; to call it ὑπάτη is to attribute to it a different one.

The study of notes, conceived as δυνάμεις, is then plainly not the same thing as the study of the sizes of intervals. The notes are not, or not merely, the boundaries of such magnitudes. How are the two studies related, and what is the role of each in harmonics as a whole? At 40.4–24, in the course of a diatribe against the thesis that the purpose of harmonic science is to develop an accurate notation, Aristoxenus commits himself to a very strong position about the role of magnitude. His remarks are most readily understood if we suppose that the notation he had in mind used signs to represent sizes of interval, rather than points of pitch. Such signs might be designed to convey instructions of the form 'Rise through the interval of a fourth', and so on. Nothing is independently known of a notation of this sort, and it is possible, though harder, to interpret his comments as directed to one in which signs represented pitches. To a modern musician this may seem more likely, but here again we have no evidence that such a notational system was ever used by Greek composers or performers. Aristoxenus is plainly thinking of a notation developed for theoretical purposes by musicologists: whatever it was, it was certainly not the 'Alypian' system in which the pitiful scrap-heap of existing scores has come down to us, since this is not open to the criticisms that Aristoxenus now deploys. The theme of his complaint is that notation is purely quantitative: hence it does not reveal the δυνάμεις of the different tetrachords, for its signs do not distinguish δυνάμεις, but only magnitudes. A grasp of the magnitudes as such, however (τὸ διαισθάνεσθαι τῶν μεγέθων αὐτῶν), is *no part* of an overall understanding of the subject (οὐδέν ἐστι μέρος τῆς συμπάσης ξυνέσεως); for through the sizes themselves (δι' αὐτῶν τῶν μεγέθων) we can gain understanding neither of the δυνάμεις of tetrachords and notes, nor of the differences between

genera, nor of the differences between composite and incomposite intervals, nor of the differences between simple and modulating sequences (τὸ ἁπλοῦν καὶ μεταβολὴν ἔχον), nor of the τρόποι (styles or genres) of melodic composition, nor indeed of anything else whatever.

This is fighting talk. We must remember that the context is polemical, and notice also that Aristoxenus is not saying that a grasp of magnitude has no part to play in harmonics, only that *by itself* (I take this to be the force of the addition of αὐτῶν to the phrases τὸ διαισθάνεσθαι τῶν μεγέθων and διὰ τῶν μεγέθων) it can give no understanding. In combination with a grasp of other data and principles, it may nevertheless make a valuable, if subordinate contribution. What seems entirely clear, however, is that propositions about intervallic sizes as such are no part of the *conclusions* that harmonics seeks to establish. If they were, a knowledge of them would necessarily be a part of τῆς συμπάσης ξυνέσεως, and it is not. If such knowledge is required of the harmonic scientist, it must therefore find its role in an earlier phase of his research rather than appearing 'as such' among his results.

Aristoxenus repeatedly affirms that harmonic science must begin from perception. It must discover by experience, not by *a priori* theorising, what intervals there are, what sequences are used, and so on, and only then proceed to enquire into the principles that explain why the facts are as they are. Thought must be applied to the data given to our hearing. It is therefore no surprise to find him asserting that it is by hearing that we judge the sizes of intervals, and by διανοία that we investigate δυνάμεις (33.6–9): this suggests that we start by setting out the ear's findings about sizes of intervals and their regular successions, and go on to interpret and explain them in terms of the δυνάμεις which thought (διανοία) reveals as their causes and principles. Propositions about sizes are data to be explained, while propositions about δυνάμεις are the principles that we seek, and that allow us to reformulate the original data in a unified and comprehensible way.

This is not to say, however, that the data given to perception cannot also include facts concerning things that are themselves conceived dynamically rather than quantitatively. On the contrary, it is precisely because perception does grasp things in their character as δυνάμεις, and because generalisation from perception reveals regularities that hold of such δυνάμεις, that we are led to consciousness of the fact that it is these regularities, and not those about magnitude as such, that constitute the significant principles of melodic 'nature'. It is on the basis of what we perceive, not of pure thought, that we are equipped to decide what sorts of truths may count as the ἀρχαί or basic principles of the science (44.11–13).

The point is well brought out in a long passage (47.8–50.14) in which Aristoxenus answers the question how a note can remain the same note when its distance from others varies. The note λιχανός, to use his example, is a ditone below the μέση in the enharmonic, a tone below the μέση in sharp diatonic, and at any of an indefinite number of positions intermediate between these in other χρόαι. Why are all these positions to be conceived as instantiating the same note? After all, the questioner continues, the distances between the fixed notes never change: would it not be better always to give a different name to a note in a different position? Notes that bound different magnitudes must surely be different notes, and notes bounding the same magnitudes, conversely, must be the same and should be given the same names.

To accept these proposals, Aristoxenus replies, would be altogether revolutionary

(μέγα τι κινεῖν). In the first place, we are aware (ὁρῶμεν) that the νήτη and μέση are different in δύναμις from the παρανήτη and λιχανός (he adds several other examples), and that is why they have different names: yet the interval between the members of each pair is the same. Differences between notes are therefore not always reflected in differences between the sizes of interval that they bound, and the fact that two intervals are the same size does not entitle us to say that the notes bounding them are the same. (We might accuse Aristoxenus of missing the point, and insist that two notes are the same if they stand at the same interval from some one specified fixed note. It could be argued in response that this thesis will meet difficulties if we go on to require that a purely quantitative account be given of the identity of the fixed note that is the reference point. But in any case it fails for a more straightforward reason: different notes often stand at the same interval from a given note. In the diatonic, for instance, the τρίτη διεζευγμένων and παρανήτη συνημμένων are both a tone and a half above the μέση; the diatonic παρυπάτη and enharmonic λιχανός are both a ditone below the μέση; and so on.)

Again, Aristoxenus continues, if we insist that every different interval implies a different musical note, we shall need a different set of names to go with every variant of the πυκνόν. But since the upper notes of the πυκνόν can be shifted through an infinite series of positions, generating different χρόαι, we shall require an infinity of names. He presumably takes this requirement to be absurd.

Finally and most significantly, if we concentrate our attention on equality and inequality of intervals, we shall throw away our discernment of genuine likeness and unlikeness (ἀποβαλοῦμεν τὴν τοῦ ὁμοίου τε καὶ ἀνομοίου διάγνωσιν). We shall be compelled, for instance, to apply the word πυκνόν to a composite interval of just one precise size, and each of the designations 'enharmonic' and 'chromatic' to just one sequence of magnitudes. But this procedure would fly in the face of τὴν τῆς αἰσθήσεως φαντασίαν, which assigns these names not by reference to unique sizes of interval, but by taking notice of the similarity existing within one form or kind (ὁμοιότητα ἑνός τινος εἴδους). In fact, the boundaries of each of the intervals constituting the items designated are limited within a τόπος (a range of variation) and are not restricted to a single position. Further, it is not physical or mathematical principles that determine the limits of these τόποι. The name πυκνόν, for instance, is to be given to any pair of intervals jointly smaller than the remainder of a fourth, not because there is anything mathematically distinct about a class so defined, but because its members all display to perception the characteristic sound of something compressed (πυκνόν), though they are unequal (ἐμφαίνεται γὰρ ἐν πᾶσι τοῖς πυκνοῖς πυκνοῦ τινος φωνὴ καίπερ ἀνίσων αὐτῶν ὄντων.) (Since πυκνά are in effect defined as pairs of intervals summing to less than 5/4 tones, the distinction between them and ἄπυκνα συστήματα corresponds roughly to our own distinction between seconds and thirds, marking the area in which concords [in the modern and not the Greek sense] shade off into discords. Notes bounding a tone or less, played simultaneously, give a 'compressed' sound: those bounding any version of a minor third or more do not.) Similarly, any system is to be called chromatic so long as it displays the chromatic character (ἕως ἂν τὸ χρωματικὸν ἦθος ἐμφαίνηται): a given genus continues to be perceived as retaining its own characteristic form of 'movement' while employing a plurality of different divisions of the tetrachord. In both cases cited, the question when structures are or are not of significantly different kinds (πυκνόν or ἄπυκνον, chromatic or enharmonic) is decided by perception (αἴσθησις).

Thus the genus, Aristoxenus goes on, remains the same while the magnitudes vary within certain limits, and while it remains the same the δυνάμεις of the notes are constant too. After all, what is there to determine whether one version of chromatic or enharmonic is the 'correct' one, rather than another? So far as αἴσθησις is concerned, the genus remains enharmonic whether the interval between the μέση and λιχανός is a ditone or some very slightly smaller interval. The εἶδος of the tetrachord is the same, and hence we must give the notes bounding the intervals the same names. (The word εἶδος is plainly not intended here as a synonym for σχῆμα, as it commonly is elsewhere in Aristoxenus: it refers to the character of the tetrachord as this is grasped by perception, the character that makes it perceptibly identifiable as an enharmonic tetrachord even in advance of any theoretical analysis.) Further, it is true in general (i.e., irrespective of genus) that so long as the names of the notes bounding a given melodic space remain the same, e.g., the μέση and ὑπάτη, so too must those of any notes falling between them, since perception always identifies those between the μέση and ὑπάτη as the λιχανός and παρυπάτη.

There is a great deal of food for thought here, but what emerges most clearly is that distinctions of δύναμις and its companion, εἶδος, are identified in the first place by perception. It is these distinctions that perception finds melodically significant, and which the harmonic scientist must seek to preserve, analyse, and organise. Differences of μέγεθος, though perceived as such and capable of 'colouring' a melody in distinguishable ways, do not create differences of melodic structure unless they correspond to differences of δύναμις. Perception distinguishes the πυκνόν from the ἄπυκνον and genus from genus: it also identifies differences of δύναμις between tetrachords and between notes, and it requires that any notes lying between designated pairs of notes, no matter at what intervals, have melodic meaning corresponding to their positions, and are to be given the appropriate dynamically significant names. Facts specified in terms of these concepts and distinctions are as much a part of the harmonic scientist's data as are those that refer to magnitudes. They are fundamental to his science because it is through their δυνάμεις, not through the mathematical sizes of intervening intervals, that notes acquire roles in melody, and it is through our perception of their δυνάμεις that we hear them as making melodic sense.

Aristoxenus attributes dynamic properties to notes, to tetrachords, and even to intervals, or in the cases of the πυκνόν and the genera, to sequences of intervals. But they are ascribed to πυκνά and genera on the basis of perceptible form (εἶδος), and to other intervals only when they are specified by reference to their particular bounding notes, never when they are identified by their sizes and nothing else. Tetrachords, too, are only said to have different δυνάμεις on occasions when they are identified by reference to the notes that form their limits. Given Aristoxenus' contention, elaborated in a passage that we considered previously (68.13–69.28), that it is δυνάμεις and not magnitudes that are the determinate features of melodic phenomena, and are therefore the harmonic scientist's proper focus of interest, it should follow that no propositions about the sizes of intervals conceived simply as such should appear either as principles or as significant conclusions of the science. We have thus returned to the position stated so uncompromisingly at 40.4–24.

A focus on δυνάμεις reveals the samenesses and differences from which melodies are created, and by reference to which their nature is to be understood. From the melodic point of view, though not from that of physics or mathematical acoustics, all

πυκνά (though some are perceptibly different from others) fall together into a class of similars; so too do all enharmonic sequences, all chromatic sequences, all diatonic sequences, and all notes of the same name—all λιχανοί, all μέσαι, and so on. These similarities are constituted by identity of δύναμις or power to determine how melody can legitimately proceed from it. Hence we may expect all important principles and conclusions of harmonics to be framed as propositions about entities of these sorts, not about the despised μεγέθη and τάσεις: they will express laws about the regular behaviour of πυκνά, of melodic genera, of tetrachords and of notes, and perhaps of some other things too, since we have no guarantee that our list is exhaustive.

Let us now return to the problems of book 3, and first to the fact that substantial amounts of Aristoxenus' reasoning presuppose what we called assumption **A**. This assumption takes the framework of fixed notes set out in section 1 as a unique and unalterable system of relations between notes of any extended series. Aristoxenus' arguments also refer, by their names, to notes other than those in the fixed framework in ways that take for granted their order, though not the sizes of the intervals by which they are separated (since these are variable). His rules apply to successions of notes lying within this complete system, and identified as elements of it. What we have now seen is that the identity of notes as ordered elements of the system constitutes their δυνάμεις (to call a note ὑπάτη, for instance, is to allude to its δύναμις), and that what we have called 'assumptions' are not intellectually constructed models or hypotheses but data derived from perception. Any sequence of notes will either be identified by perception as instantiating a segment of the system, or it will be rejected as unmelodic. To hear a sequence of pitches as a melody or a part of one, is not merely to notice that its elements stand in certain definite relations of pitch. It involves hearing this note as constituting the λιχανός, that one as the παρυπάτη, and these intuitions imply further expectations about the δυνάμεις of the notes associated with them, or capable of being so associated. If this note is the λιχανός, for example, there can be no note above it nearer than the μέση.

The existence of the framework of fixed notes, forming the boundaries of conjunct and disjunct tetrachords, ensures that some properties of notes, dynamically conceived, can be represented quantitatively. If this note is the παραμέση, the μέση must stand a tone below it, and the ὑπάτη μεσῶν a fourth below that: such propositions remain true no matter what shifts of genus and shade (χρόα) occur. What is important is that this rule is expressed as a truth about the δύναμις of a note, the παραμέση, not as a generalisation about successions of quantified intervals as such. Many such truths, as we have seen, cannot be stated in quantitative terms at all, and even in cases like the present one the references to magnitudes can be eliminated. We may take it as a datum that notes in equivalent positions in successive tetrachords are concordant with one another (it is this principle that appears in a quantitative form as **L**). To be the παραμέση is, in part, to be the lower bounding note of a tetrachord with which the tetrachord in succession (ἑξῆς) below it does not share a note. It follows at once, as Aristoxenus shows in the opening propositions of book 3, that the note immediately below the παραμέση (i.e., the μέση) is separated from it by a tone, and that the παραμέση stands a fifth above the lower boundary of the tetrachord below it. These quantitative truths are consequences of dynamic ones, not the other way about. They are to be conceived as corollaries to analyses of the δυνάμεις of identifiable notes.

It is in connection with the other major problem of book 3, the form of argument

that appears most frequently in applications of **T** and **D**, that the focus on notes and δυνάμεις rather than on intervals and their sizes pays the greatest interpretative dividends. Let us consider such an argument once again.

A tone cannot stand immediately below a ditone. If it did, since the upper boundary of a tone is the lower boundary of a πυκνόν (by **T**), and the lower boundary of a ditone is the upper boundary of a πυκνόν (by **D**), and since *ex hypothesi* the two boundaries coincide, one πυκνόν will be ἑξῆς with another, which is melodically impossible (by **L**). If this argument (or indeed **T** itself) is conceived in terms of rules about successions of intervals, it becomes almost impenetrable. The two πυκνά cannot be present if the sequence of tone and ditone is. If we interpret the rules as saying that a πυκνόν is always an available option below a ditone and above a tone, then it will follow that if we proceed downwards through a ditone we may continue to a πυκνόν, and if we proceed upwards through a tone we may continue to a πυκνόν. But nothing in this entails that if we proceed upwards through the first of these πυκνά the second is available as a continuation, since we have precisely *not* reached the shared note through the interval of a tone by which the upper πυκνόν was implied.

If, however, we interpret the argument as being based on a conception of identifiable notes and their δυνάμεις, the obscurities vanish. To be an identifiable note is to have a specific role in the system and to stand to other notes in specific relations, some of which are quantifiable. These relations express the note's δύναμις, its nature conceived as a power to determine the ways in which a proper melodic sequence can proceed from it. The powers of any given note are not independent of one another: they comprise a coherent unity constituting what the note is, coherent in the sense that the structure of the system requires these powers to form aspects of a single melodic role or δύναμις. In the light of this approach, rule **D** entails that if a note's role requires that a melodic sequence rising to the next note above must jump through an incomposite ditone, then it is also such as to require a πυκνόν below itself. **T** entails that any note having the power to propel a downwards melodic sequence through the disjunctive interval is also such as to require a πυκνόν above itself. Thus, **D** and **T** each express a coherent δύναμις capable of attaching to different notes. But they cannot both belong to the same note, since that note's nature would then be incoherent, one feature of it being inconsistent with another. It cannot be within the δύναμις of a single note to admit a πυκνόν both above itself and below, for such a δύναμις cannot exist within the framework of the system in which δυνάμεις are defined. The rules of melodic succession may be conceived as elucidations of the bundles of δυνάμεις that cohere in notes by virtue of their roles in the system, and as setting limits to the combinations of δυνάμεις that can fit together to constitute any one note. The law appealed to most often, **L**, is no more than a handy summary of some of the most general constraints upon δυνάμεις that the organisation of the system imposes. Though it is possible, contrary to what Aristoxenus says, for notes with different δυνάμεις to fall on the same pitch, it is not possible for one note to possess a δύναμις which is demonstrably incoherent.

The difficulties involved in treating **T** and **D** as rules about successions of intervals no longer arise. It is unnecessary to argue as if sequences of πυκνά were somehow present in intervals that do not contain them, or as if the intervals implying them could do so when they are not present themselves; and we are no longer committed to the arbitrary rule that implied and actual intervals must be capable of occurring in every

sequential combination that is notionally available. Instead we have the simple rule that a note cannot manifest its power inconsistently, that is, in such a way that it cannot be a single element in the melodic system. A note is a unified whole, whose character is expressed in the ordered set of relations with all other melodic elements. A hypothetical note whose character cannot be coherently expressed in this way is not an element of any legitimate melody, that is, of a sequence which αἴσθησις can interpret as making melodic sense.

The notes of the system thus constitute a complex of dynamic relations, of possibilities for melodic movement. The laws of harmonics express these possibilities: some, but not all, can be stated quantitatively, whereas all can be stated in terms of melodic function. The theorems of book 3 take quantification as far as it can be pressed, but it has limits on which Aristoxenus is right to insist, and the quantitative rules themselves are aspects of laws holding primarily between elements of the dynamic order.

At the core of the theory of melodic δύναμις is the idea that in order to be part of a melody, a sound must have a determinate role in a structural system within whose forms perception may grasp and interpret it. The melodic relations between one note and another are not constituted or wholly determined by their relative pitches, but by the intersection of their potentialities and implications within such a structure: that is what creates the possibility of melodic sense, and provides a basis for distinguishing sense from nonsense. The fact, for example, that the interval between one note and another is a tone is not what gives the notes the status of elements in a melody, or generates implications for melodically possible continuations. As Aristoxenus says, many notes, conceived as δυνάμεις, can stand in the same relation to the same size of interval. One may proceed upwards by a tone from the diatonic παρυπάτη, or from the sharp diatonic λιχανός, or from the μέση in any of the genera, or from the ὑπάτη in the tonic chromatic. But melodically speaking, one is not doing the same thing in all these cases, and different laws of continuation will apply. The melody may be moving from the second to the third note of a tetrachord, or from the third to the fourth, or rising through the interval of disjunction from the top of one tetrachord to the bottom of another, or proceeding in a single step through an interval which is melodically composite or divisible in that form of scale into two semitones. Again, as we have seen, there are occasions when movement through a tone is melodically inadmissible. The tone is not as such a melodic interval: it is a possible instantiation of the dynamic relation between some pairs of elements in the system, and not of that between others.

Two sounds may have the same structural role, the same δύναμις, and determine what are structurally equivalent possibilities for continuation, while not standing at the same distance from other given notes. The λιχανός may lie a tone below the μέση, or a ditone, or anywhere between these limits: but the laws of harmonics, in their most unified and illuminating form, concern such intervals as 'the interval between the μέση and λιχανός', rather than specific sizes. This feature of Greek harmonic theory, with its reliance on the concept of 'movable' notes, is commonly supposed to be alien to more modern Western practice, but this is a mistake: difference of degree has been mistaken for difference in kind. A note functions as the third of the scale of C minor, for instance, not by being a pitched sound whose vibration-rate stands to that of C in a certain definite ratio: given a suitable melodic context, a suitable implicit structure, any of quite a wide variety of pitches will be heard as E♭, and will count as filling its melodic

role. This fact is well known, but it is often misconstrued by being interpreted as an illustration of the ear's capacity to ignore inaccuracy and approximation, to treat what is roughly or nearly E♭ as though it 'really' were E♭. It is nothing of the sort, since the notion of a 'real' E♭ is the merest chimera, a monster born of musicologists' too hasty embrace of propositions belonging to physical acoustics. The third of the scale of C minor is a unified bundle of determinate melodic functions or senses, and there is no reason whatever to believe that there is some one ideal E♭ that transmits this sense better than all others. Of course different musical ears, with different experience and training, will prefer some candidates to others (though what they prefer may not be the same in every C minor context), but talk of 'perfect' intonation, as conceivably pursued by a string-player or a singer, should not mislead us. The string-player's ideal intonation is different from that of a well-tuned piano: orchestral violinists and devotees of unaccompanied choral music often find the piano's temperament disturbing. But what this shows is only that the functions of certain notes are being 'shaded' or 'coloured' by one performer in ways that another finds incongruous. It does nothing to imply that only one of these χρόαι is correct.

Aristoxenus' remarks at 23.1–22 are to be read in just this sense. He claims that the form of melodic composition (μελοποιία) within the enharmonic genus that uses a true ditone between the μέση and λιχανός, rather than something slightly smaller, is one of the most admirable, though most people cannot tolerate it. Their ears are accustomed to the 'sweetness' of chromatic melodies, so that when performing in the enharmonic they tend to raise the λιχανός somewhat towards its chromatic locus. They could be brought to an understanding of the excellence of the form that employs the true ditone by a thorough training in ancient styles. These comments express an aesthetic preference, and trace its source to people's familiarity with one kind of music rather than another. But Aristoxenus carefully refrains from asserting that an exact ditone is the only *correct* interval between the μέση and the enharmonic λιχανός: it is not, and elsewhere he says this explicitly and with emphasis (49.10–18).

There is also little difficulty in adapting Aristoxenus' warnings about notation (39.4–41.24) to fit a modern context, though the type he had in mind, whatever it was, was certainly very different from our own. The staff notation with which we are familiar is in fact the product of an uneasy compromise between quantitative and dynamic principles, but it is only too easy to be misled by it into supposing that relations between notes are distances on the continuum of pitch, and nothing else, and that the task of the performer is to get these distances exactly 'right'.

If we accept that melody is to be understood against a background of functional or dynamic relations, the fact that the same role may be performed equally well, though perhaps with different emotional effect, by any of a variety of pitches, need not be in the least mysterious. Many similar systems of significance exist. We have a formalised system against which we understand the movements of a driver's hand as signals: because this is so, any of a great diversity of visible patterns of movement may count as signalling 'I am about to turn right', and each (within limits) is as good as any other. Our understanding of spoken language depends on our capacity to treat as functionally equivalent any of a set of sound-types within a wide range of variation: if it were not so, no educated New Yorker could talk to his cab-driver let alone to an Englishman. There are limits, of course, though vague ones, but that does not mean that there is an ideal, a pure and correct form (a Platonic Form?) of pronunciation to

which all others more or less adequately approximate. The Queen's English, if that means the English spoken by the Queen of England, is just one rather esoteric χρόα among many.

Melodic relations have more to do with mathematics than do those of spoken language or of drivers' signals. One reason for this, on Aristoxenus' view, is that certain fundamental melodic relations are in fact determined to a magnitude that has, at the most, only a minute and imperceptible range of variation (55.3–6). He is talking here about the concords (fourth, fifth, and octave), and it is true that they are cases where the ear will not grant functional equivalence to relations that differ beyond the limits of a rather small region. (Even here, however, the amount of variation tolerated or even deliberately used in practice by competent musicians is far from negligible.) The profusion of quantitative conclusions among Aristoxenus' theorems is due in part to the fact that the framework of the system is arranged around concords—relations, that is, which have the melodic role of concords and are heard as such in their context, whatever their precise magnitudes may be. This is a dynamic and not a quantitative consideration: it yields quantitative results only because of Aristoxenus' conviction that the sizes of intervals that are heard as concords are, as a matter of fact, to all intents and purposes determinate. The other source of quantitative conclusions is Aristoxenus' adoption of certain divisions of the tetrachord as paradigmatic examples: the quantitative results extracted from their use must therefore be interpreted *only* as exemplary, never as fixing an ideal.

Melody exists within a structure, the interrelations of whose parts create melodic form and significance. To enunciate the 'laws' of melody is to explicate this structure, and the ways in which its elements interlock to form patterns of potentiality and implication. Such laws have little to do with those of physics or acoustics, and can certainly not be derived either from them or from models of mathematical perfection. In taking this position Aristoxenus has the great advantage of being right. Where his stance seems more vulnerable is in its assumption that there is only one such system, and that everything with a claim to be called a melody must be disqualified if it implies relations that this unique structure cannot incorporate. We may agree to the application of normative rules within a given system of function and significance, while admitting that there can be others equally coherent. Aristoxenus' attitude is not, I think, the result of mere prejudice: it stems from his consciously and carefully thought out approach to the history of music, a topic that the present paper cannot even begin to investigate. It is remarkable, for all that, how well the basic conceptions of Aristoxenus' system have stood the test of time within the Western tradition. The resources of 'harmony', in the modern sense, and more recently of chromaticism, atonality and the rest, have enlarged our conceptions of music, but they have done little to restructure the patterns within which we understand a sequence of sounds as a melody. Not all our tunes obey all Aristoxenian laws, but his system has been modified and elaborated rather than buried.

By way of a postscript, it is worth remarking that if my suggestions are even approximately correct, they cast doubt on the cogency of the arguments levelled against Aristoxenus by the shrewdest of his critics in antiquity. Ptolemy (*Harm.* 19.16–21.20) attacks Aristoxenus' general approach to harmonics on various grounds, but particularly for treating intervals, rather than notes, as the fundamental

determinants of melodic order. Like the Pythagoreans, Ptolemy construes notes as magnitudes whose relations are to be expressed as ratios, whereas Aristoxenus displays them as dimensionless points on a linear continuum of pitch, separated by distances. For Aristoxenus it is therefore only the distances or intervals to which magnitudes can be attached. But these, Ptolemy argues, cannot be responsible for the phenomena studied by harmonics. They are just so much empty space, mere distance, and yet the Aristoxenians behave as if they were bodies (σώματα) and the notes themselves bodiless (ἀσώματα): the truth is quite the reverse (20.5–9, 21.9–20). Ptolemy believes that in Aristoxenian theory it is intervals and not notes that interact and relate with one another to produce melody, and that it is to regularities in the behaviour of intervals, quantitatively conceived, that the laws of harmonics, as understood by this school, are applied. But it is notes, he insists, not the mere gaps between them, that are the stuff from which melodies can be built.

It is characteristic of Ptolemy, however, that he wishes to construe all significant harmonic relations in quantitative terms, and to find the explanation and the real essence of harmonic regularities in principles that belong to mathematics. His understanding of the proposition that 'hearing is the criterion in harmonics in respect of matter and qualifications, reason (λόγος) in respect of form and cause' (*Harm*. 3.3–5) is utterly un-Aristoxenian. It is not surprising, then, that he argues as if Aristoxenus' generalisations about intervallic magnitudes were intended to be autonomous principles, since he can find in Aristoxenus no more fundamental conceptions of a quantitative sort from which these rules are capable of being derived. He shows no sign of having understood the Aristoxenian notion of δύναμις or even of having noticed its relevance. If he had, he would have realised that for Aristoxenus, just as much as in his own system, the basic determinants of order, and the entities whose properties and behaviour the laws of harmonics describe, are notes, not intervals. These notes are not Ptolemaic or Pythagorean magnitudes: but since from the quantitative point of view they are dimensionless points between magnitudes, Ptolemy's mathematical mind has sprung to the conclusion that this is *all* that they are, so convicting itself of the fallacy engagingly stigmatised by Medawar as 'nothing-buttery'.[6] What a note is, in addition to that, is an entity of a specifically musical kind, whose dynamic properties can be fully expressed only in terms of melodic, not mathematical relations. In Aristoxenus' view it is a grotesque error to suppose that the laws of harmonics can be derived from or explained by principles extrinsic to harmonics itself or translated out of the autonomous language of melodic analysis into terminology drawn from mathematics or physics or any other such field (*Harm. Elem.* 32.18–28, 44.15–18). The concepts applicable to melody, the forms under which its nature (φύσις) is to be described and explained, must be understood in their own terms, through their relations to one another. The characters and powers of the notes, which are the elements of melody, can be discovered only by a trained musician who is prepared to grasp them in the way his educated ear dictates (see 32.10–34.30), as implications and potentialities for movement within the dynamically ordered framework of the harmonic system. The physicist has nothing to offer: the mathematician is needed, if at all, only to do a few elementary sums.[7]

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NOTES

[1] The best modern edition is *Aristoxeni Elementa Harmonica* ed. R. da Rios (Rome, 1954, in the series *Scriptores Graeci et Latini consilio Academiae Lynceorum editi*), which includes an Italian translation with explanatory notes, and a collection of the fragments. *The Harmonics of Aristoxenus* ed. H.S. Macran (Oxford: Clarendon, 1902, reprinted by Georg Olms Verlag, Hildesheim and New York, 1974) has a good text and a valuable commentary: the English translation should be read with some caution. References in the present paper are by Meibom's pages and lines, which are indicated in all modern texts.

[2] Unlike the Pythagoreans, Aristoxenus has no qualms about dividing the interval of a tone into equal parts. The thesis that this is mathematically impossible arises from a representation of intervals as numerical ratios of magnitudes which is foreign to Aristoxenus' harmonics. See, e.g., (Euclid) *Sectio Canonis* prop. 16, cf. 18.

[3] That these ἀπορίαι are students' questions was suggested to me in correspondence by R.P. Winnington-Ingram, whose generous interest in my work I should like, once again, to acknowledge. Perhaps the expressions of surprise and lack of comprehension that Aristoxenus occasionally records elsewhere (e.g. 60.19, 68.14) also reflect students' reactions to his 'seminars'.

[4] Ps.-Plutarch *De Musica* 1135b, in a passage certainly derived from Aristoxenus, argues that a proposed analysis of a certain ancient scale-form must be wrong, on the grounds that it places two ditones in sequence. The author explicitly points out that one of the ditones in question is composite, yet he does not hesitate to apply the rule. In a sense, he is entitled to do so, since a sequence of this sort is bound to conflict with **L** so long as at least one of the ditones is incomposite. But this cannot be the basis of the criticism here, since **L** would also disqualify the analysis that the author goes on to approve. **L**, perhaps, is not to be applied to sequences belonging to music of the remote past. But in that case, whatever the grounds of the rule he employs may be, they are certainly quite different from those of *Harm. Elem.*

[5] This sentence is not in the MSS, but has been restored by Marquard and subsequent editors on the basis of the parallel at 70.29–33: the restoration is undoubtedly correct.

[6] P.B. Medawar, review of Pierre Teilhard de Chardin, *The Phenomenon of Man*, *Mind* 70(1961), reprinted in *The Art of the Soluble* (London: Methuen, 1967) and in *Pluto's Republic* (Oxford: Oxford University Press, 1982). Nothing-buttery, Medawar goes on in another delicious phrase, is 'always part of the minor symptomatology of the bogus'. There is nothing bogus about Ptolemy, of course: he simply does not look closely enough at the work he is criticising.

[7] My thanks are due to Alan C. Bowen for his valuable comments on a draft of this paper. He has rescued me from a number of mistakes, and has indicated various points at which clarification was needed. I have tried to respond to his suggestions: several, however, raise fundamental issues which cannot be resolved without a great deal of further investigation, and this must wait for another occasion.